Output Process of Variable Bit-Rate Flows in On-chip Networks Based on Aggregate Scheduling

Fahimeh Jafari, Axel Jantsch, and Zhonghai Lu
Royal Institute of Technology (KTH), Sweden
Email: {fjafari, axel, zhonghai}@kth.se

***Abstract*—This paper proposes an approach for more accurate analyzing of output flows in FIFO multiplexing on-chip networks with aggregate scheduling by considering peak behavior of flows. The key idea of our proposed method involves presenting and proving a technical proposition to derive output arrival curve for an individual flow under the mentioned system model.**

I. Introduction

Since the number of real-time communication services being deployed on NoCs is increasing [1], it is clear that architectures based on aggregate scheduling, which schedule multiple flows as an aggregate flow, will be an appropriate option for transmitting real-time traffic. For example, the composition of flows sharing the same buffer can be considered as an aggregate flow [2]. Furthermore, real-time applications require stringent QoS guarantees which usually employed by tight performance bounds. As analyzing output behavior of flows gives an exact vision about output metrics used for obtaining performance bounds, we aim for deriving the output characterization of Variable Bit-Rate (VBR) traffic transmitted in the FIFO order and scheduled as aggregate. In this paper, based on network calculus [3][4], we present and prove the required proposition for calculating output arrival curve under the mentioned system model.

The VBR is a class of traffic in which the rate can vary significantly from time to time, containing bursts. Real-time VBR flows can be characterized by a set of four parameters, (L, p, σ, ρ), where L is the maximum transfer size, p peak rate, σ burstiness, and ρ average sustainable rate [4]. Our assumption is that the application-specific nature of the network enables to characterize traffic with sufficient accuracy.

Authors in [5] present a theorem for calculating per-flow output arrival curve in tandem networks of rate-latency nodes traversed by leaky-bucket shaped flows. This theorem investigates computing output traffic characterization only for average behavior of flows while the proposed proposition in this paper considers both average and peak behavior, which results in a more accurate analysis.

II. Network Calculus Background

Network Calculus is a theory that provides deep analysis on flow problems encountered in networking. It uses the abstraction of service curve to model a network element processing traffic flows modeled with an arrival curve in terms of input and output flow relationships. Network elements such as routers, links, and regulators, can be modeled by corresponding service curves. A flow f is an infinite stream of unicast traffic (packets) sent from a source node to a destination node. To model the average and peak characteristics of a flow, Traffic SPECification (TSPEC) is used. As shown in Fig. 1, with TSPEC, f is characterized by an *arrival curve* $\alpha(t) = min(L + pt, \sigma + \rho t)$ in which $p \geq \rho$ and $\sigma \geq L$.

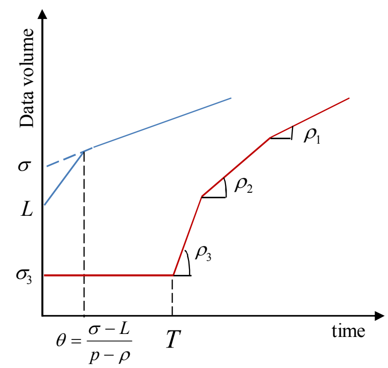

Fig. 1. Left Curve is the arrival curve of flow f with TSPEC (L, p, σ, ρ) and right one is the pseudoaffine service curve with three leaky-bucket stages

Theorem 1. *(Output Flow [4]). Assume a flow, constrained by arrival curve α, traverses a system that offers a service curve of β, the output flow is constrained by the arrival curve $\alpha^* = \alpha \oslash \beta$, where $\oslash$ represents the min-plus deconvolution of two functions $f, g \in F, (f \oslash g)(t) = sup_{s \geq 0} \{f(t + s) - g(s)\}$.*

III. Analysis

We assume that flows are classified into a pre-specified number of aggregates at their source nodes. In addition, we assume that traffic of each aggregate is buffered and transmitted in the FIFO order and different aggregates are buffered separately. The network is lossless, and packets traverse the network using a deterministic routing.

we first consider a class of curves, namely pseudoaffine curves [5], which is a multiple affine curve shifted to the right and given by $\beta = \delta_T \otimes [\otimes_{1 \leq x \leq n} \gamma_{\sigma_x, \rho_x}]$. In fact, a pseudoaffine curve represents the service received by single flows in tandems of FIFO multiplexing rate-latency nodes. Due to concave affine curves, it can be rewritten as $\beta = \delta_T \otimes [\wedge_{1 \leq x \leq n} \gamma_{\sigma_x, \rho_x}]$, where the non-negative term T is denoted as *offset*, and the affine curves between square brackets as leaky-bucket stages. Fig. 1 shows a pseudoaffine service curve with three leaky-bucket stages.

We now propose the proposition for computing output arrival curve as follows.

Proposition 1. *(Output Arrival Curve with FIFO) Consider a VBR flow, with TSPEC (L, p, ρ, σ), served in a node that guarantees to the flow a pseudo affine service curve equal to $\beta = \delta_T \otimes \gamma_{\sigma_x,\rho_x}$. The output arrival curve α^* given by:*

$$\alpha^* = \begin{cases} \theta > T & \gamma_{(p\wedge R)T+\theta(p-R)^+ + L-\sigma_x, p\wedge R} \\ & \wedge\gamma_{\sigma-\sigma_x+\rho T,\rho} \\ \theta \leq T & \gamma_{\sigma-\sigma_x+\rho T,\rho} \end{cases} \tag{1}$$

where $\wedge$ represents the minimum operation.

Proof. *From Theory 1, the output flow is constrained by the arrival curve $\alpha^* = \alpha \oslash \beta = sup_{u\geq 0}\{\alpha(t+u) - \beta(u)\}$. Thus, $\alpha^* = sup_{u\geq 0}\{min(\sigma + \rho(t+u), L + p(t+u)) - \sigma_x - \rho_x(u-T)^+\}$*

We now consider two different situations including $\theta \leq T$ and $\theta > T$. If $\theta \leq T$, we have:

$$\begin{aligned}
\alpha^* &= sup_{u\geq 0}\{min(\sigma + \rho(t+u), L + p(t+u)) - \sigma_x \\
&\quad -\rho_x(u-T)^+\} \\
&= sup_{0\leq u\leq T}\{min(\sigma + \rho(t+u), L + p(t+u)) - \sigma_x\} \\
&\quad \vee sup_{u>T}\{min(\sigma + \rho(t+u), L + p(t+u)) \\
&\quad -\sigma_x - \rho_x u + \rho_x T\} \\
&= \{min(\sigma + \rho(t+T), L + p(t+T)) - \sigma_x\} \vee \\
&\quad sup_{u>T}\{min(\sigma + \rho(t+u), L + p(t+u)) - \sigma_x \\
&\quad -\rho_x u + \rho_x T\} \\
&= \{\sigma + \rho(t+T) - \sigma_x\} \vee sup_{u>T}\{\sigma + \rho(t+u) - \sigma_x \\
&\quad -\rho_x u + \rho_x T\} \\
&= \{\sigma + \rho(t+T) - \sigma_x\} \vee sup_{u>T}\{\sigma + \rho t + \rho_x T - \sigma_x \\
&\quad +u(\rho - \rho_x)\}
\end{aligned}$$

Since $\rho \leq \rho_x$ and thus $\rho - \rho_x$ is negative, u in the second term should get its lowest possible value to achieve supremum. Thus, we have

$$\begin{aligned}
&= \{\sigma + \rho(t+T) - \sigma_x\} \vee \{\sigma + \rho(t+T) - \sigma_x\} \\
&= \sigma + \rho(t+T) - \sigma_x = \gamma_{\sigma-\sigma_x+\rho T,\rho}
\end{aligned} \tag{2}$$

If $\theta > T$, we have:

$$\begin{aligned}
\alpha^* &= sup_{u\geq 0}\{min(\sigma + \rho(t+u), L + p(t+u)) - \sigma_x - \\
&\quad \rho_x(u-T)^+\} \\
&= sup_{0\leq u\leq T}\{min(\sigma + \rho(t+u), L + p(t+u)) - \sigma_x\} \\
&\quad \vee sup_{u>T}\{min(\sigma + \rho(t+u), L + p(t+u)) - \sigma_x \\
&\quad -\rho_x u + \rho_x T\} \\
&= \{min(\sigma + \rho(t+T), L + p(t+T)) - \sigma_x\} \vee sup_{u>T}\{ \\
&\quad min(\sigma + \rho(t+u) - \sigma_x - \rho_x u + \rho_x T, L + p(t+u) \\
&\quad -\sigma_x - \rho_x u + \rho_x T)\}
\end{aligned} \tag{3}$$

For completing the proof, we need to consider the second term in right side of Eq. (3) in details. Therefore, we call it $Term_2$ in the following:

$$\begin{aligned}
Term_2 = &sup_{u>T}\{min(\sigma + \rho(t+u) - \sigma_x - \rho_x u + \rho_x T, \\
&L + p(t+u) - \sigma_x - \rho_x u + \rho_x T)\}
\end{aligned}$$

For solving $Term_2$, we consider two different situations including $t + u \leq \theta$ and $t + u \geq \theta$. Thus, if $t + u \geq \theta$, we have $u > T$ and $t + u \geq \theta$.

$$\begin{aligned}
\Rightarrow Term_2 &= sup_{u>T}(\sigma + \rho(t+u) - \sigma_x - \rho_x u + \rho_x T) \\
&= sup_{u>T}(\sigma + \rho t + \rho_x T - \sigma_x + (\rho - \rho_x)u) \\
&= \sigma + \rho t + \rho_x T - \sigma_x + (\rho - \rho_x)T \\
&= \sigma + \rho(t+T) - \sigma_x = \gamma_{\sigma-\sigma_x+\rho T,\rho}
\end{aligned} \tag{4}$$

If $t + u \leq \theta$, we have $u > T$ and $t + u \leq \theta \Rightarrow u \leq \theta - t$.

$$\begin{aligned}
\Rightarrow Term_2 &= sup_{T<u\leq\theta-t}(L + p(t+u) - \sigma_x - \rho_x u + \rho_x T) \\
&= sup_{T<u\leq\theta-t}(L + pt + \rho_x T - \sigma_x + (p - \rho_x)u)
\end{aligned}$$

Selecting an appropriate value for u depends on if $(p - \rho_x)$ is positive or negative. Therefore, we have two different situations including $p > \rho_x$ and $p \leq \rho_x$. If $p > \rho_x \Rightarrow (p - \rho_x)$ is positive and u should be the highest possible value to have supremum value. Thus, due to $u = \theta - t$, $Term_2 = L + \rho_x(t+T) - \sigma_x + \theta(p - \rho_x)$. If $p \leq \rho_x \Rightarrow (p - \rho_x)$ is negative. Therefore, u gets its lowest value and $Term_2$ is equal to $L + p(t+T) - \sigma_x$.

$$\Rightarrow Term_2 = L + (p \wedge \rho_x)(t+T) - \sigma_x + \theta(p - \rho_x)^+ \tag{5}$$

From Eq. 3, 4 and 5, if $\theta > T$, we have:

$$\begin{aligned}
\alpha^* &= min\Big(L + (p \wedge \rho_x)(t+T) - \sigma_x + \theta(p - \rho_x)^+, \\
&\quad \sigma + \rho(t+T) - \sigma_x)) \\
&= \gamma_{(p\wedge R)T+\theta(p-R)^+ + L-\sigma_x, p\wedge R} \wedge \gamma_{\sigma-\sigma_x+\rho T,\rho}
\end{aligned} \tag{6}$$

From Eq. 2 and 6, we straightforwardly obtain the thesis.

IV. Conclusions

Real-time applications exert stringent requirements on networks. To this end, we have presented and proved the required proposition for computing the output arrival curve of VBR flows in a FIFO multiplexing network to detail output traffic characterization. The proposition can be applied for an architecture based on aggregate scheduling. In the future, we will present a formal approach to calculate performance bounds under the mentioned system model.

References

[1] Z. Shi, A. Burns, L. S. Indrusiak, "Schedulability Analysis for Real Time On-Chip Communication with Wormhole Switching", *International Journal of Embedded and Real-Time Communication Systems*, vol. 1, no. 2, pp. 1-22, 2010.

[2] Y. Qian, Z. Lu, W. Dou, "Analysis of Worst-case Delay Bounds for Best-effort Communication in Wormhole Networks on Chip", *In the Proceedings of the 3rd ACM/IEEE International Symposium on Networks-on-Chip (NOCS'09)*, pp. 44-53, San Diego, CA, 2009.

[3] C. Chang, *Performance Guarantees in Communication Networks*, Springer-Verlag, 2000.

[4] J. Y. L. Boudec and P. Thiran, *Network Calculus: A Theory of Deterministic Queuing Systems for the Internet*, (LNCS, vol. 2050). Berlin, Germany: Springer-Verlag, 2004.

[5] L. Lenzini, L. Martorini, E. Mingozzi, and G. Stea, "Tight end-to-end per-flow delay bounds in fifo multiplexing sink-tree networks", *Performance Evaluation*, vol. 63, no. 9, pp. 956-987, 2006.