

Communication Performance in Network-on-Chips

Axel Jantsch

Royal Institute of Technology, Stockholm

November 24, 2004



Overview

Introduction

Communication Performance

Organizational Structure

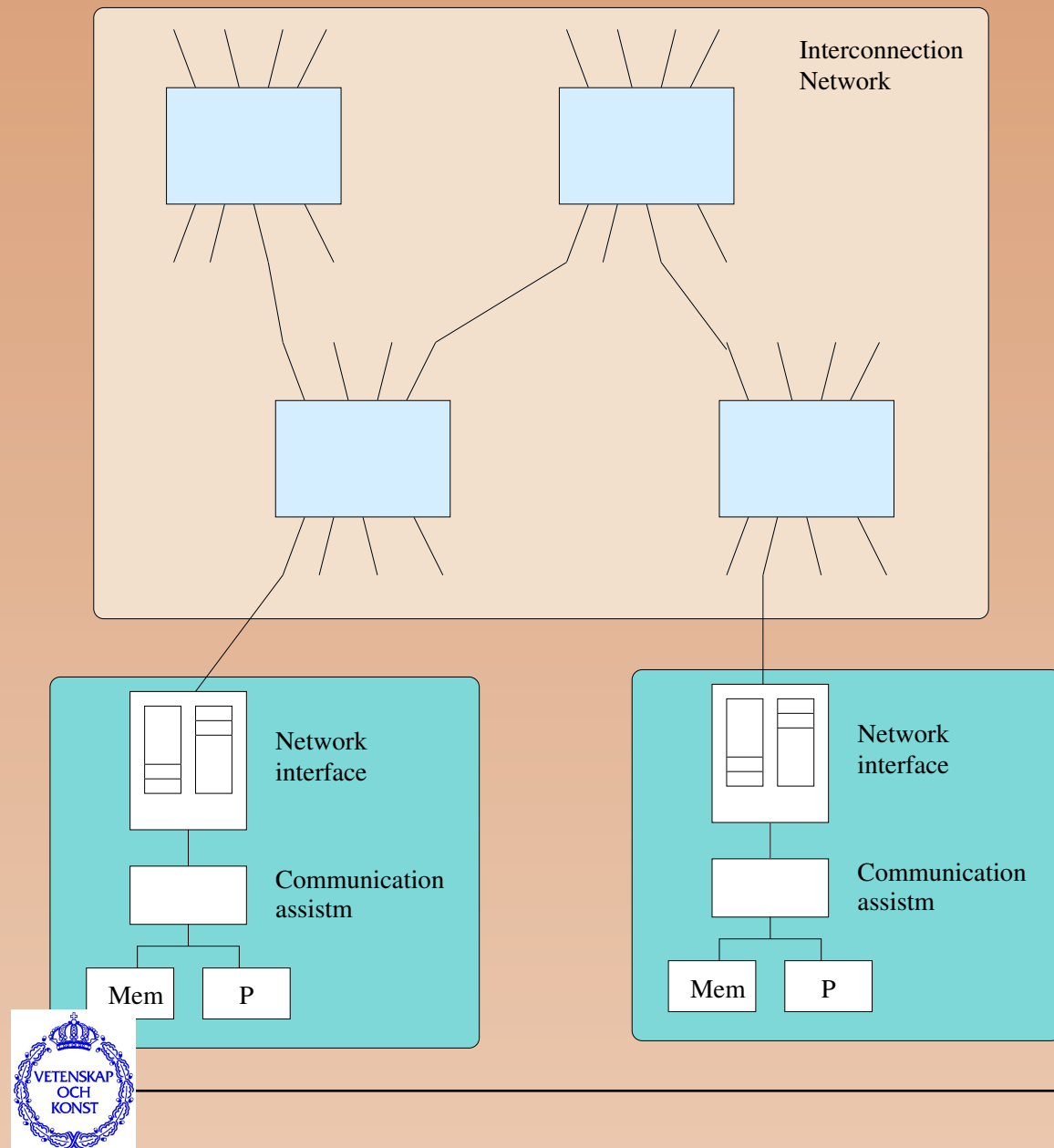
Interconnection Topologies

Trade-offs in Network Topology

Routing



Introduction



- **Topology:** How switches and nodes are connected
- **Routing algorithm:** determines the route from source to destination
- **Switching strategy:** how a message traverses the route
- **Flow control:** Schedules the traversal of the message over time

Basic Definitions



Basic Definitions

Message is the basic communication entity.

Flit is the basic flow control unit. A message consists of 1 or many flits.

Phit is the basic unit of the physical layer.



Basic Definitions

Message is the basic communication entity.

Flit is the basic flow control unit. A message consists of 1 or many flits.

Phit is the basic unit of the physical layer.

Direct network is a network where each switch connects to a node.

Indirect network is a network with switches not connected to any node.



Basic Definitions

Message is the basic communication entity.

Flit is the basic flow control unit. A message consists of 1 or many flits.

Phit is the basic unit of the physical layer.

Direct network is a network where each switch connects to a node.

Indirect network is a network with switches not connected to any node.

Hop is the basic communication action from node to switch or from switch to switch.



Basic Definitions

Message is the basic communication entity.

Flit is the basic flow control unit. A message consists of 1 or many flits.

Phit is the basic unit of the physical layer.

Direct network is a network where each switch connects to a node.

Indirect network is a network with switches not connected to any node.

Hop is the basic communication action from node to switch or from switch to switch.

Diameter is the length of the maximum shortest path between any two nodes measured in hops.

Routing distance between two nodes is the number of hops on a route.

Average distance is the average of the routing distance over all pairs of nodes.



Basic Switching Techniques

Circuit Switching A real or virtual circuit establishes a direct connection between source and destination.



Basic Switching Techniques

Circuit Switching A real or virtual circuit establishes a direct connection between source and destination.

Packet Switching Each packet of a message is routed independently. The destination address has to be provided with each packet.



Basic Switching Techniques

Circuit Switching A real or virtual circuit establishes a direct connection between source and destination.

Packet Switching Each packet of a message is routed independently. The destination address has to be provided with each packet.

Store and Forward Packet Switching The entire packet is stored and then forwarded at each switch.



Basic Switching Techniques

Circuit Switching A real or virtual circuit establishes a direct connection between source and destination.

Packet Switching Each packet of a message is routed independently. The destination address has to be provided with each packet.

Store and Forward Packet Switching The entire packet is stored and then forwarded at each switch.

Cut Through Packet Switching The flits of a packet are pipelined through the network. The packet is not completely buffered in each switch.



Basic Switching Techniques

Circuit Switching A real or virtual circuit establishes a direct connection between source and destination.

Packet Switching Each packet of a message is routed independently. The destination address has to be provided with each packet.

Store and Forward Packet Switching The entire packet is stored and then forwarded at each switch.

Cut Through Packet Switching The flits of a packet are pipelined through the network. The packet is not completely buffered in each switch.

Virtual Cut Through Packet Switching The entire packet is stored in a switch only when the header flit is blocked due to congestion.



Basic Switching Techniques

Circuit Switching A real or virtual circuit establishes a direct connection between source and destination.

Packet Switching Each packet of a message is routed independently. The destination address has to be provided with each packet.

Store and Forward Packet Switching The entire packet is stored and then forwarded at each switch.

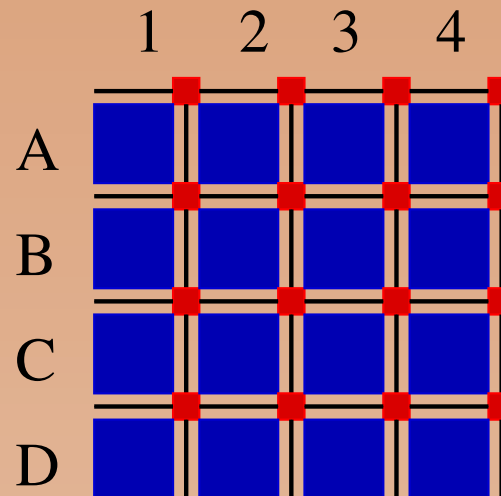
Cut Through Packet Switching The flits of a packet are pipelined through the network. The packet is not completely buffered in each switch.

Virtual Cut Through Packet Switching The entire packet is stored in a switch only when the header flit is blocked due to congestion.

Wormhole Switching is cut through switching and all flits are blocked on the spot when the header flit is blocked.



Latency



$$\text{Time}(n) = \text{Admission} + \text{ChannelOccupancy} + \text{RoutingDelay} + \text{ContentionDelay}$$

Admission is the time it takes to emit the message into the network.

ChannelOccupancy is the time a channel is occupied.

RoutingDelay is the delay for the route.

ContentionDelay is the delay of a message due to contention.



Channel Occupancy

$$\text{ChannelOccupancy} = \frac{n + n_E}{b}$$

n ... message size in bits

n_E ... envelop size in bits

b ... raw bandwidth of the channel



Routing Delay

Store and Forward:

$$T_{sf}(n, h) = h\left(\frac{n}{b} + \Delta\right)$$

n ... message size in bits

n_p ... size of message fragments in bits

h ... number of hops

b ... raw bandwidth of the channel

Δ ... switching delay per hop



Routing Delay

Store and Forward:

$$T_{sf}(n, h) = h\left(\frac{n}{b} + \Delta\right)$$

Circuit Switching:

$$T_{cs}(n, h) = \frac{n}{b} + h\Delta$$

n ... message size in bits

n_p ... size of message fragments in bits

h ... number of hops

b ... raw bandwidth of the channel

Δ ... switching delay per hop



Routing Delay

Store and Forward:

$$T_{sf}(n, h) = h\left(\frac{n}{b} + \Delta\right)$$

Circuit Switching:

$$T_{cs}(n, h) = \frac{n}{b} + h\Delta$$

Cut Through:

$$T_{ct}(n, h) = \frac{n}{b} + h\Delta$$

n ... message size in bits

n_p ... size of message fragments in bits

h ... number of hops

b ... raw bandwidth of the channel

Δ ... switching delay per hop



Routing Delay

Store and Forward:

$$T_{sf}(n, h) = h\left(\frac{n}{b} + \Delta\right)$$

Circuit Switching:

$$T_{cs}(n, h) = \frac{n}{b} + h\Delta$$

Cut Through:

$$T_{ct}(n, h) = \frac{n}{b} + h\Delta$$

Store and Forward with
fragmented packets:

$$T_{sf}(n, h, n_p) = \frac{n - n_p}{b} + h\left(\frac{n_p}{b} + \Delta\right)$$

n ... message size in bits

n_p ... size of message fragments in bits

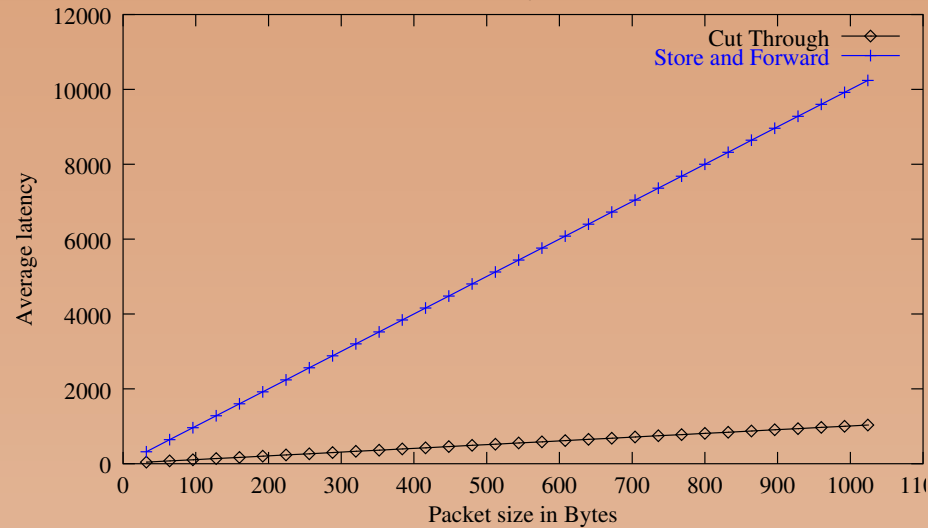
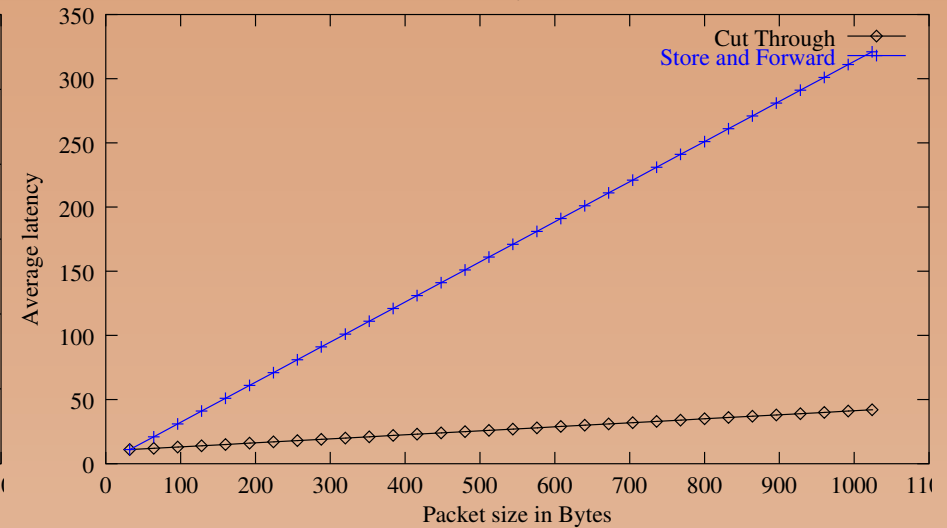
h ... number of hops

b ... raw bandwidth of the channel

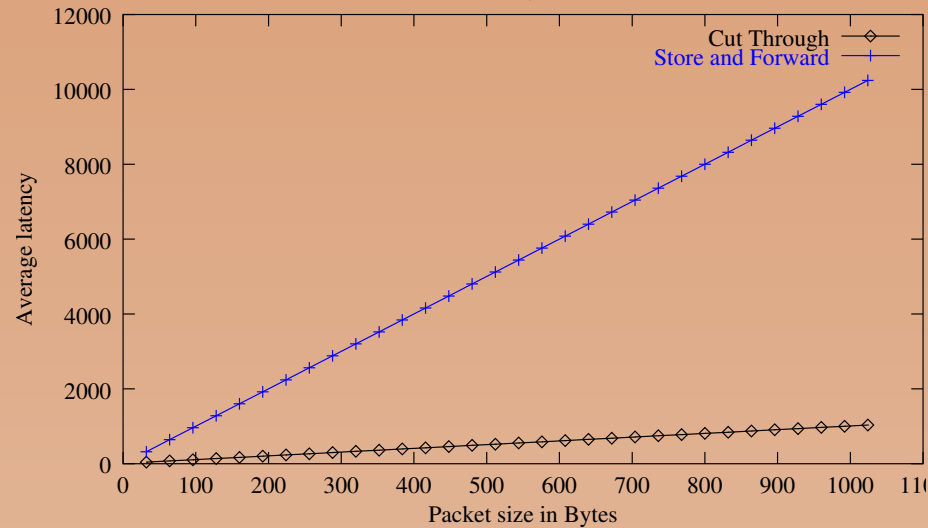
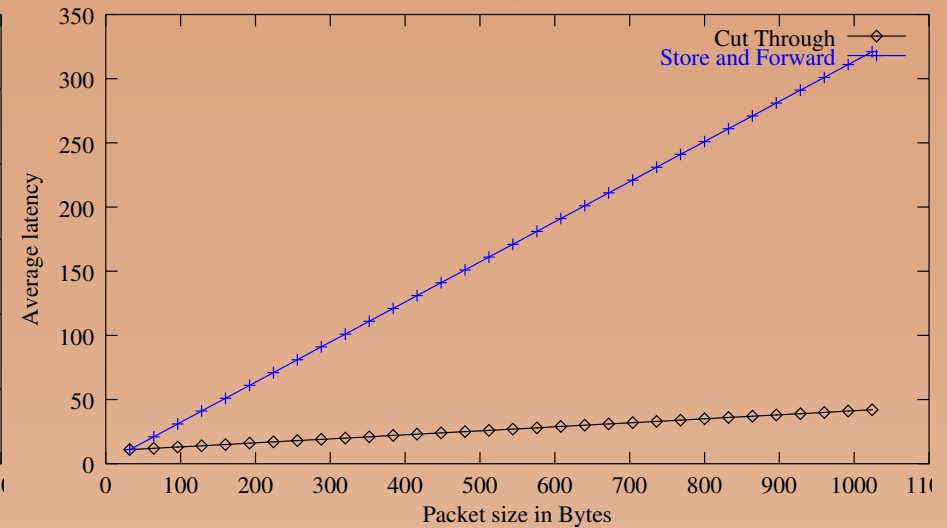
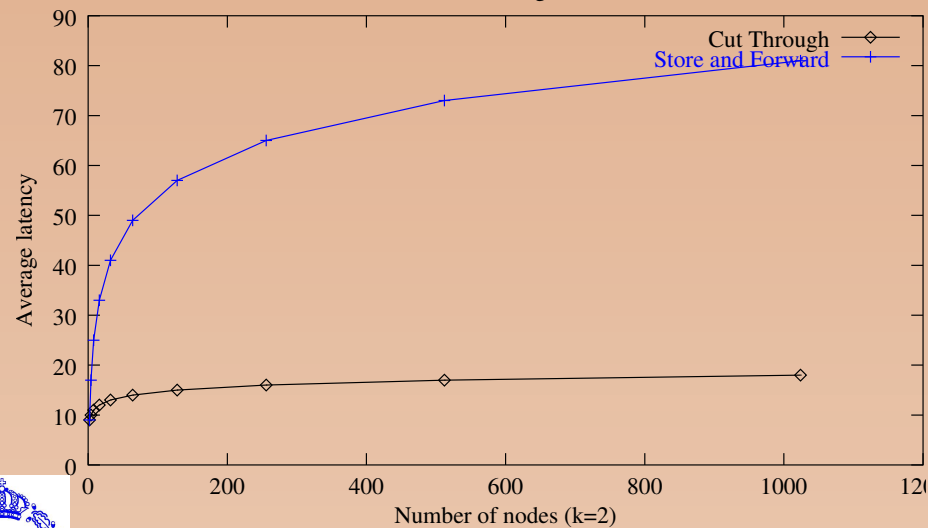
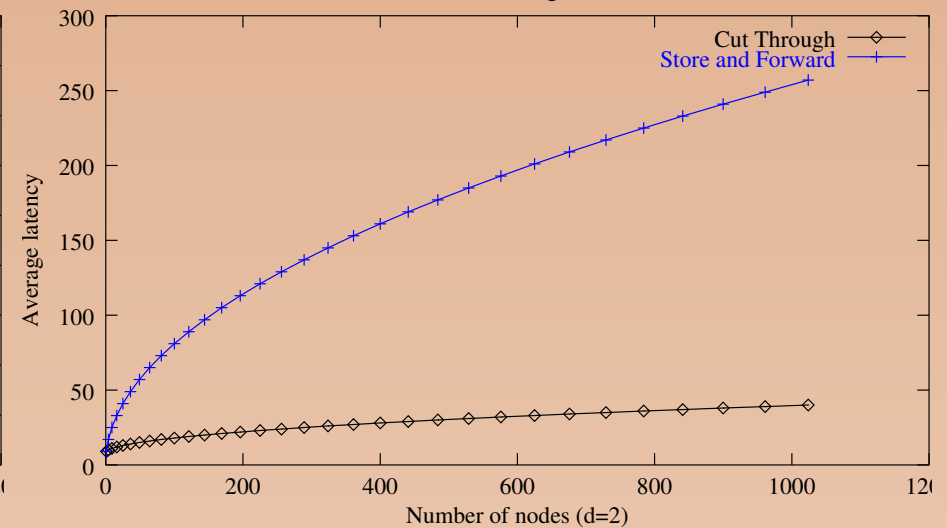
Δ ... switching delay per hop



Routing Delay: Store and Forward vs Cut Through

SF vs CT switching; $d=2$, $k=10$, $b=1$ SF vs CT switching; $d=2$, $k=10$, $b=32$ 

Routing Delay: Store and Forward vs Cut Through

SF vs CT switching; $d=2$, $k=10$, $b=1$ SF vs CT switching; $d=2$, $k=10$, $b=32$ SF vs CT switching, $k=2$, $m=8$ SF vs CT switching, $d=2$, $m=8$ 

Local and Global Bandwidth

$$\begin{aligned}
 \text{Local bandwidth} &= b \left(\frac{n}{n + n_E + w\Delta} \right) \\
 \text{Total bandwidth} &= Cb[\text{bits/second}] = Cw[\text{bits/cycle}] = C[\text{phits/cycle}] \\
 \text{Bisection bandwidth} &\dots \text{ minimum bandwidth to cut the net into two equal parts.}
 \end{aligned}$$

b ... raw bandwidth of a link;

n ... message size;

n_E ... size of message envelope;

w ... link bandwidth per cycle;

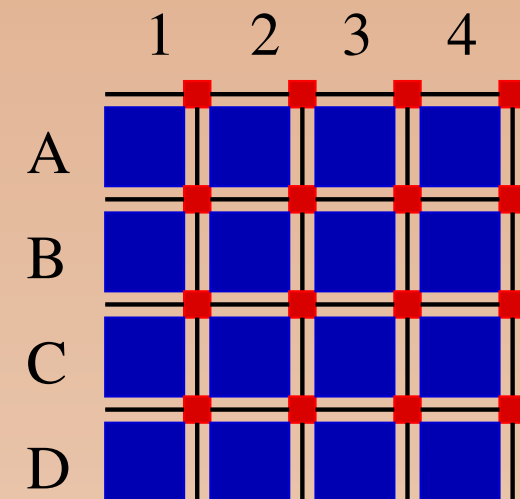
Δ ... switching time for each switch in cycles;

$w\Delta$... bandwidth lost during switching;

C ... total number of channels;

For a $k \times k$ mesh with bidirectional channels:

$$\begin{aligned}
 \text{Total bandwidth} &= (4k^2 - 4k)b \\
 \text{Bisection bandwidth} &= 2kb
 \end{aligned}$$



Link and Network Utilization

total load on the network: $L = \frac{Nhl}{M} [\text{phits/cycle}]$

load per channel: $\rho = \frac{Nhl}{MC} [\text{phits/cycle}] \leq 1$

M ... each host issues a packet every M cycles

C ... number of channels

N ... number of nodes

h ... average routing distance

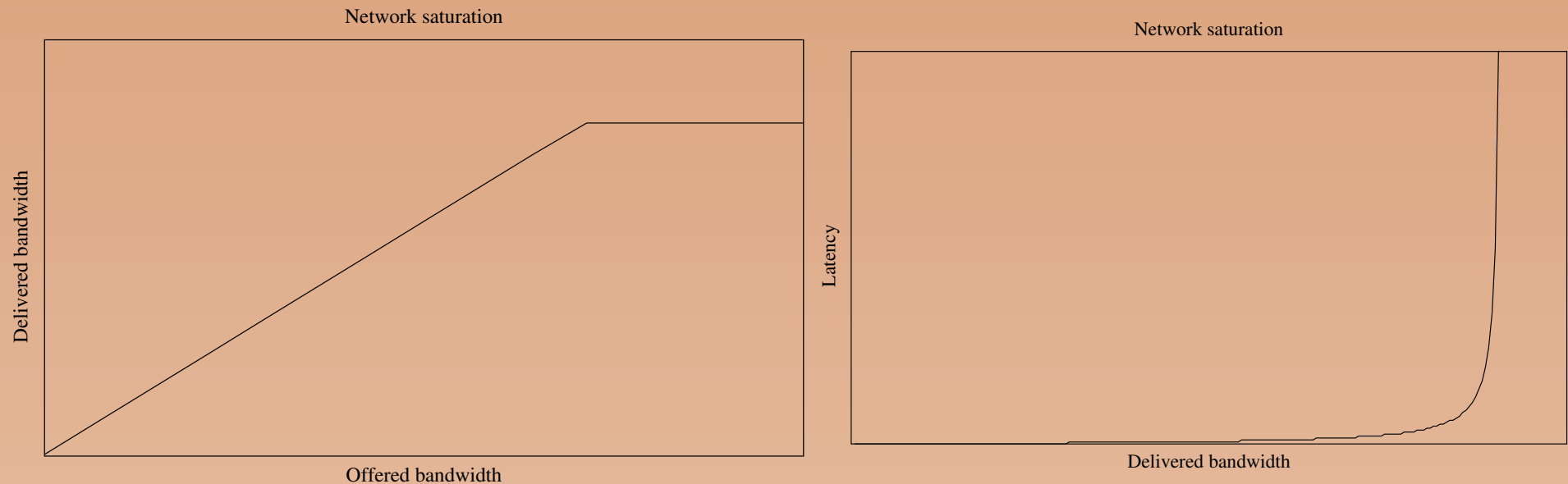
$l = n/w$... number of cycles a message occupies a channel

n ... average message size

w ... bandwidth per channel



Network Saturation



Typical saturation points are between 40% and 70%.
The saturation point depends on

- Traffic pattern
- Stochastic variations in traffic
- Routing algorithm

Organizational Structure

- Link
- Switch
- Network Interface



Link

Short link At any time there is only one data word on the link.

Long link Several data words can travel on the link simultaneously.

Narrow link Data and control information is multiplexed on the same wires.

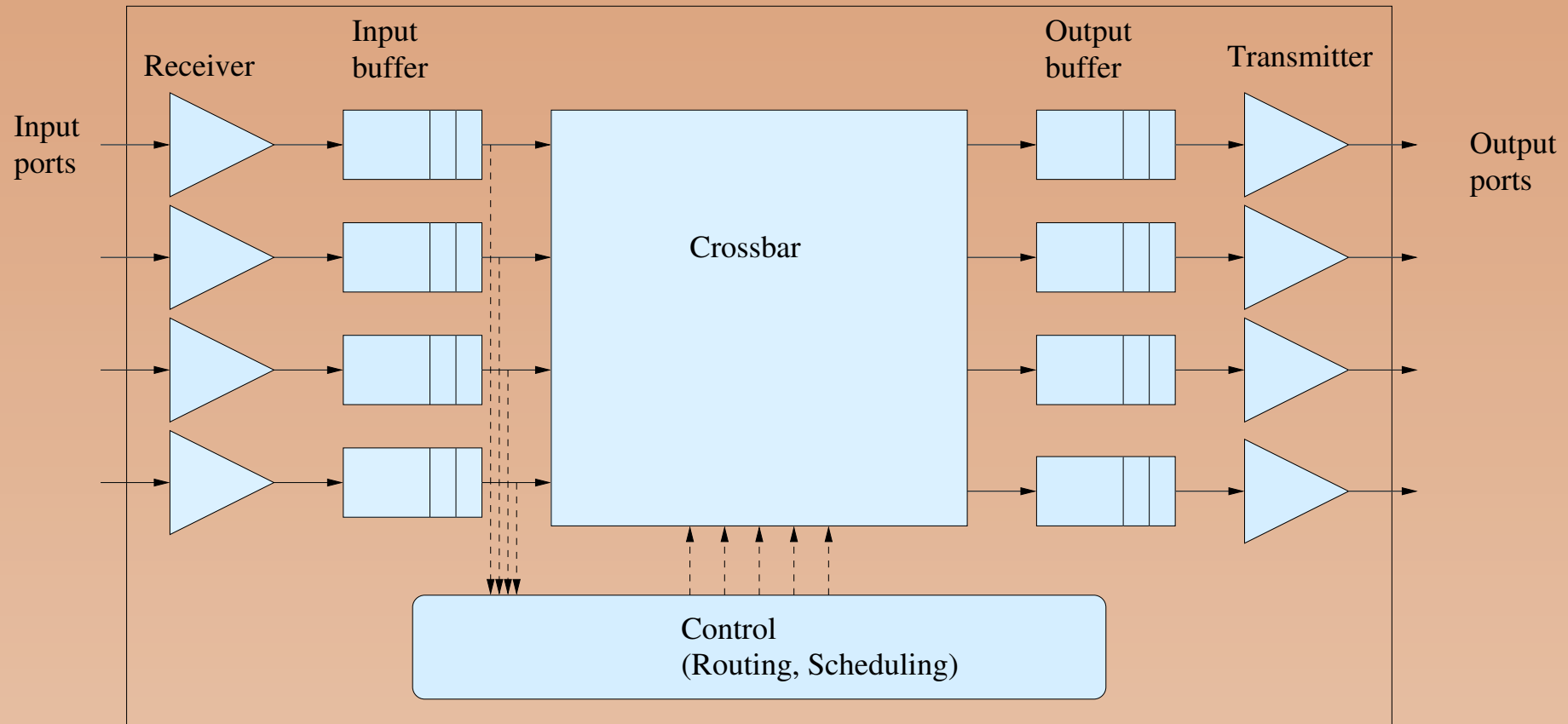
Wide link Data and control information is transmitted in parallel and simultaneously.

Synchronous clocking Both source and destination operate on the same clock.

Asynchronous clocking The clock is encoded in the transmitted data to allow the receiver to sample at the right time instance.



Switch



Switch Design Issues

Degree: number of inputs and outputs;

Buffering

- Input buffers
- Output buffers
- Shared buffers

Routing

- Source routing
- Deterministic routing
- Adaptive routing

Output scheduling

Deadlock handling

Control flow



Network Interface

- Admission protocol
- Reception obligations
- Buffering
- Assembling and disassembling of messages
- Routing
- Higher level services and protocols

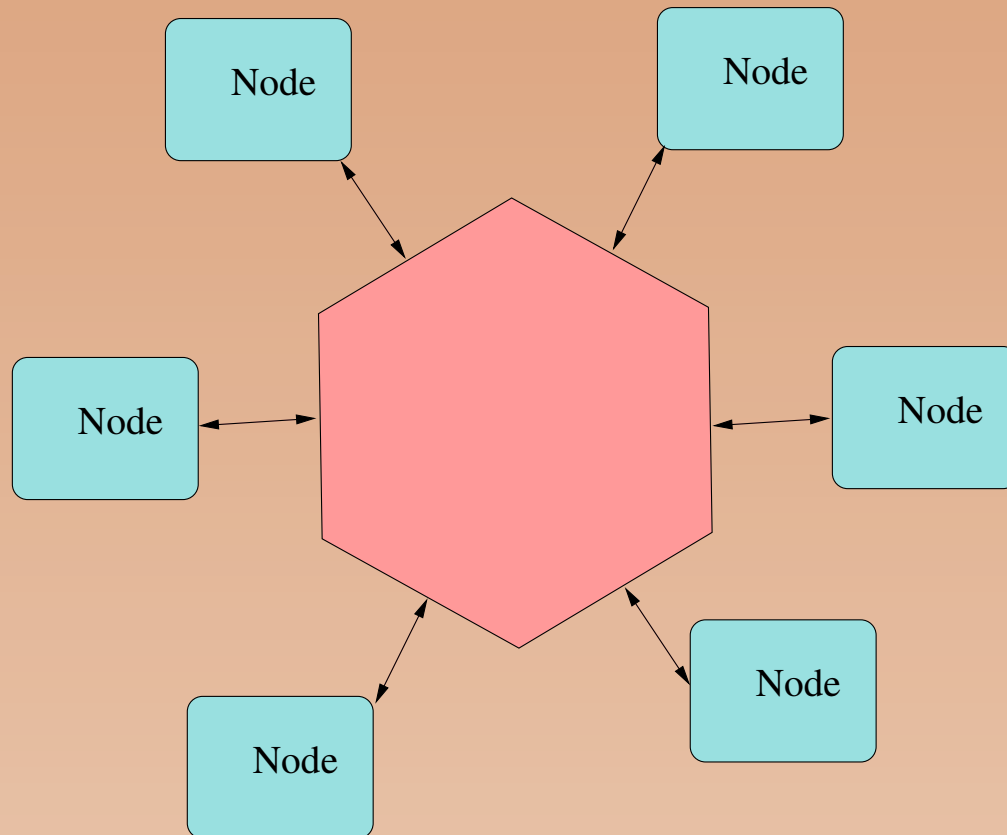


Interconnection Topologies

- Fully connected networks
- Linear arrays and rings
- Multidimensional meshes and tori
- Trees
- Butterflies



Fully Connected Networks



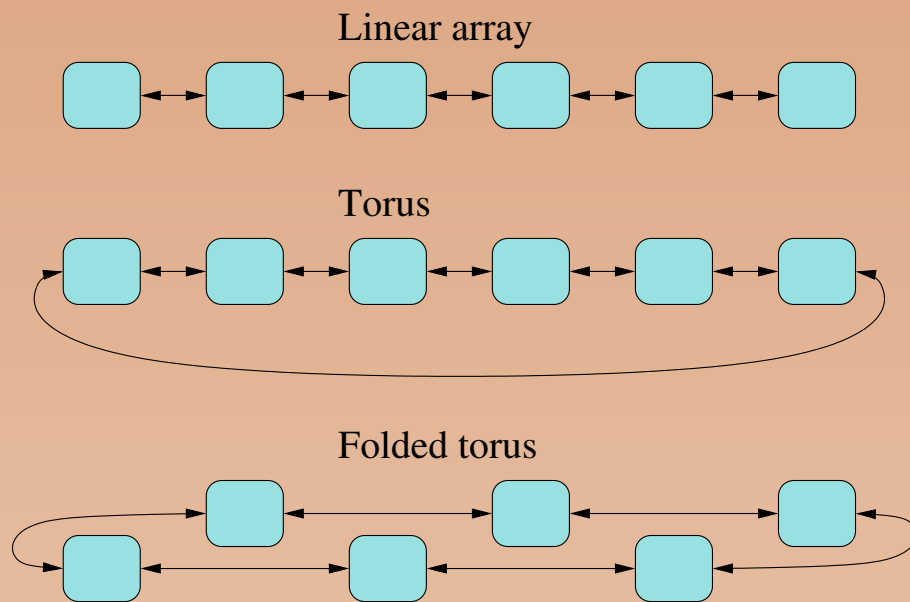
Bus:

switch degree	=	N
diameter	=	1
distance	=	1
network cost	=	$O(N)$
total bandwidth	=	b
bisection	=	b
bandwidth		

Crossbar:

switch degree	=	N
diameter	=	1
distance	=	1
network cost	=	$O(N^2)$
total bandwidth	=	Nb
bisection	=	Nb
bandwidth		

Linear Arrays and Rings



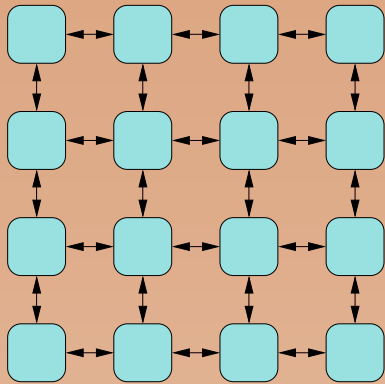
Linear

array:	switch degree	=	2
	diameter	=	$N - 1$
	distance	\sim	$2/3N$
	network cost	=	$O(N)$
	total bandwidth	=	$2(N - 1)b$
	bisection	=	$2b$
	bandwidth		

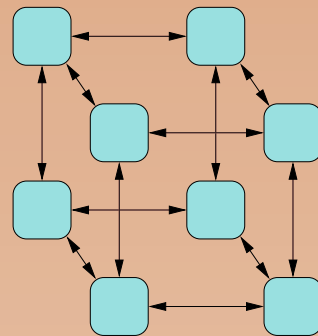
Torus:	switch degree	=	2
	diameter	=	$N/2$
	distance	\sim	$1/3N$
	network cost	=	$O(N)$
	total bandwidth	=	$2Nb$
	bisection	=	$4b$
	bandwidth		

Multidimensional Meshes and Tori

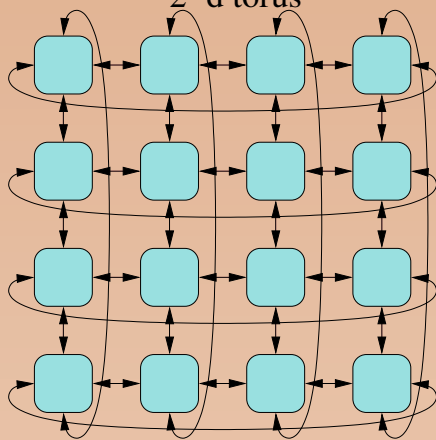
2-d mesh



3-d cube



2-d torus



k -ary d -cubes are d -dimensional tori with unidirectional links and k nodes in each dimension:

$$\text{number of nodes } N = k^d$$

$$\text{switch degree} = d$$

$$\text{diameter} = d(k - 1)$$

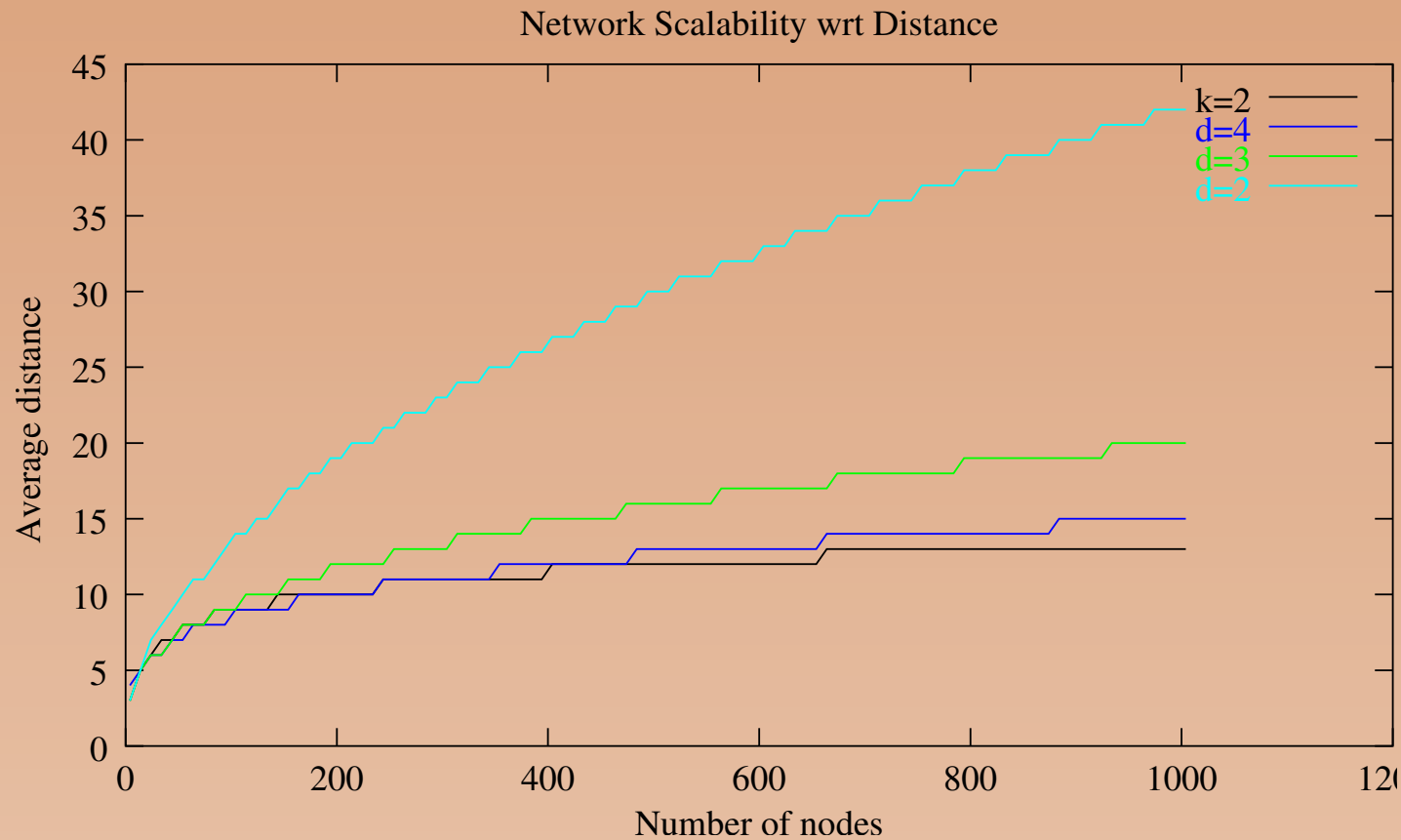
$$\text{distance} \sim d_2^1(k - 1)$$

$$\text{network cost} = O(N)$$

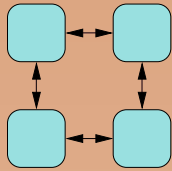
$$\text{total bandwidth} = 2Nb$$

$$\text{bisection bandwidth} = 2k^{(d-1)}b$$

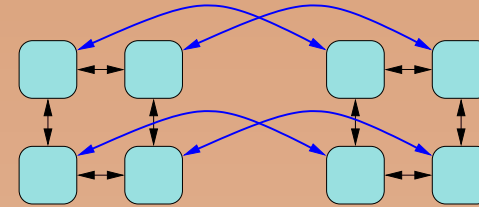
Routing Distance in k -ary n -Cubes



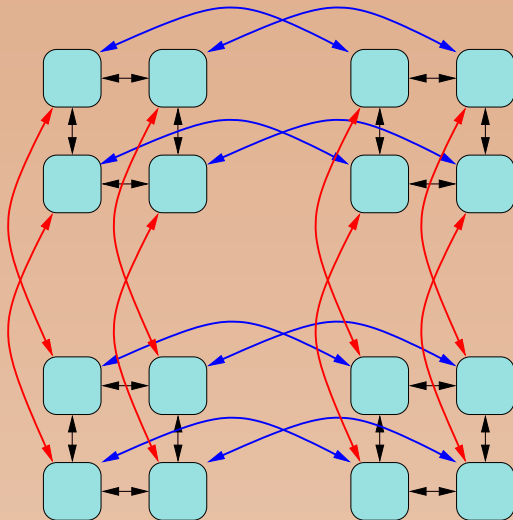
Projecting High Dimensional Cubes



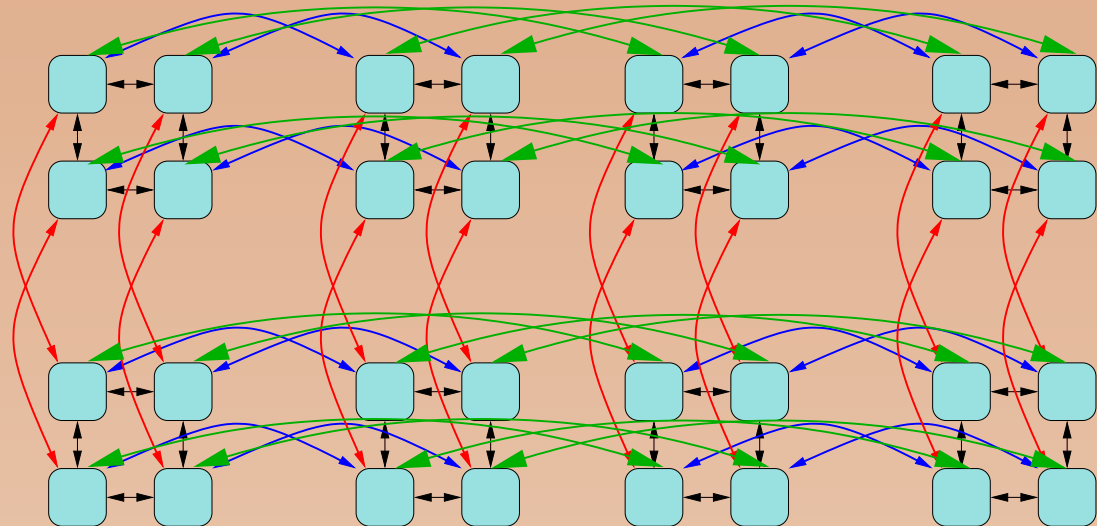
2-ary 2-cube



2-ary 3-cube

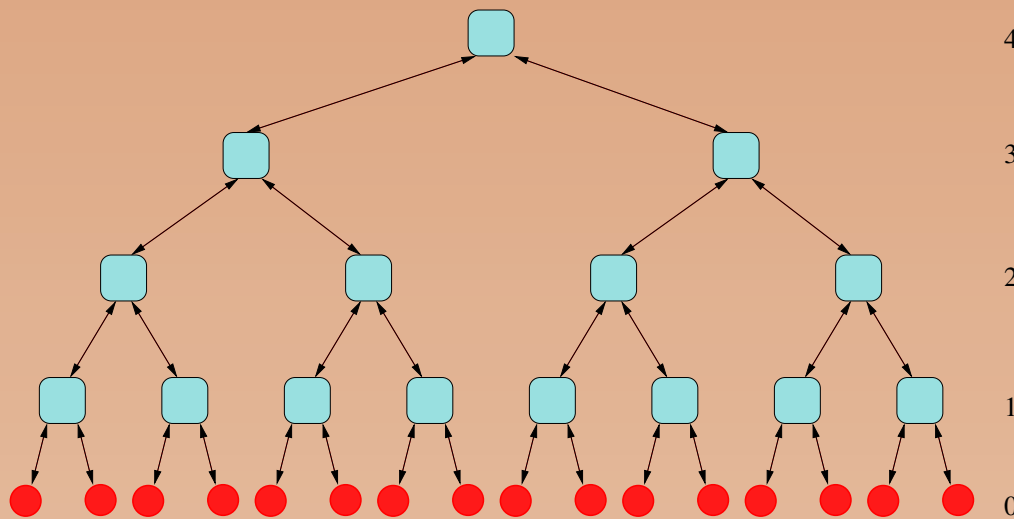


2-ary 4-cube



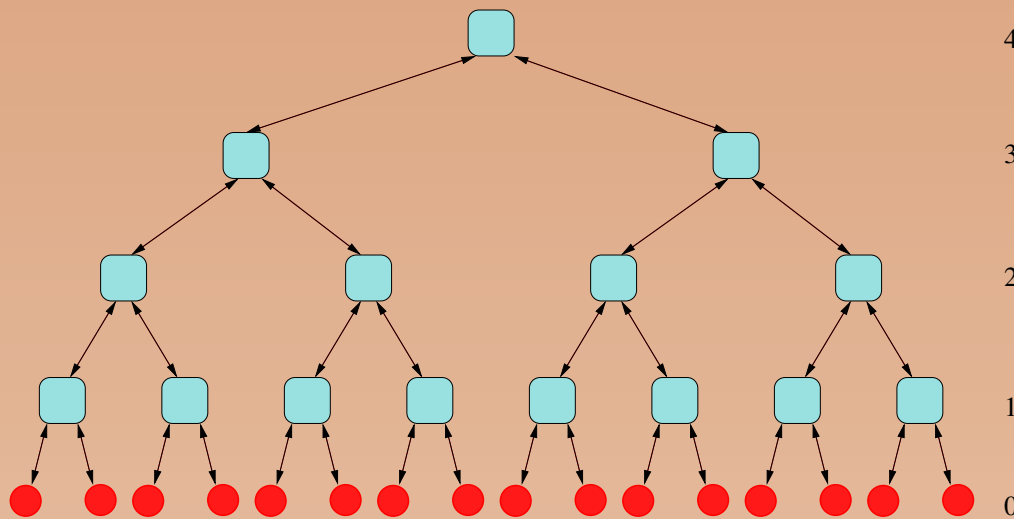
2-ary 5-cube

Binary Trees



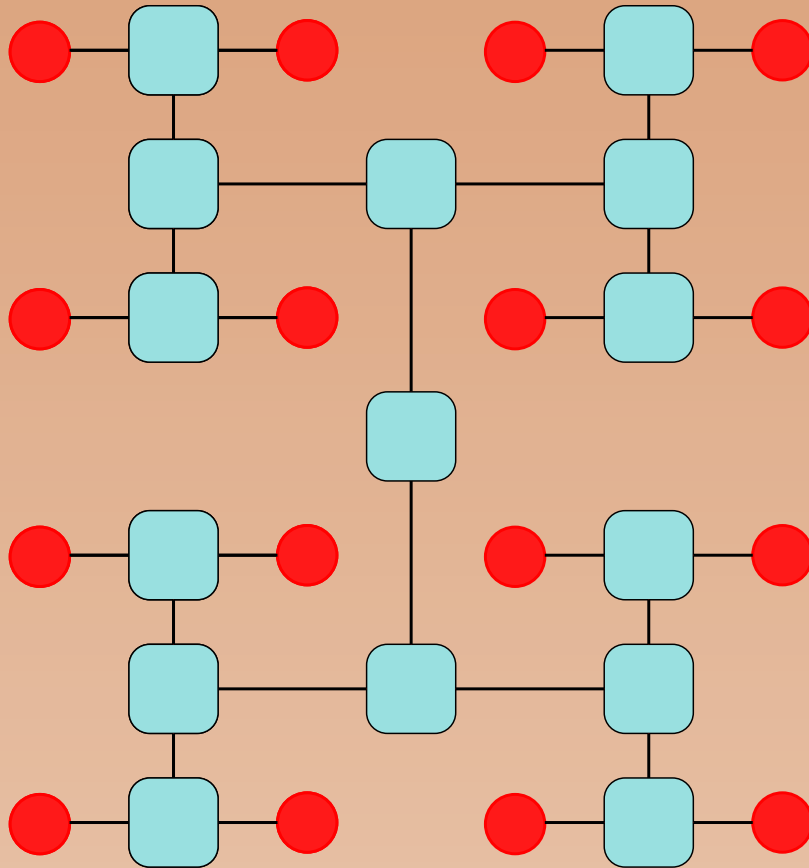
number of nodes N	$=$	2^d
number of switches	$=$	$2^d - 1$
switch degree	$=$	3
diameter	$=$	$2d$
distance	\sim	$d + 2$
network cost	$=$	$O(N)$
total bandwidth	$=$	$2 \cdot 2(N - 1)b$
bisection bandwidth	$=$	$2b$

k -ary Trees



4	number of nodes N	$=$	k^d
3	number of switches	\sim	k^d
2	switch degree	$=$	$k + 1$
1	diameter	$=$	$2d$
0	distance	\sim	$d + 2$
	network cost	$=$	$O(N)$
	total bandwidth	$=$	$2 \cdot 2(N - 1)b$
	bisection bandwidth	$=$	kb

Binary Tree Projection



- Efficient and regular 2-layout;
- Longest wires in resource width:

$$lW = 2^{\lfloor \frac{d-1}{2} \rfloor - 1}$$

d	2	3	4	5	6	7	8	9	10
N	4	8	16	32	64	128	256	512	1024
lW	0	1	1	2	2	4	4	8	8

k -ary n -Cubes versus k -ary Trees

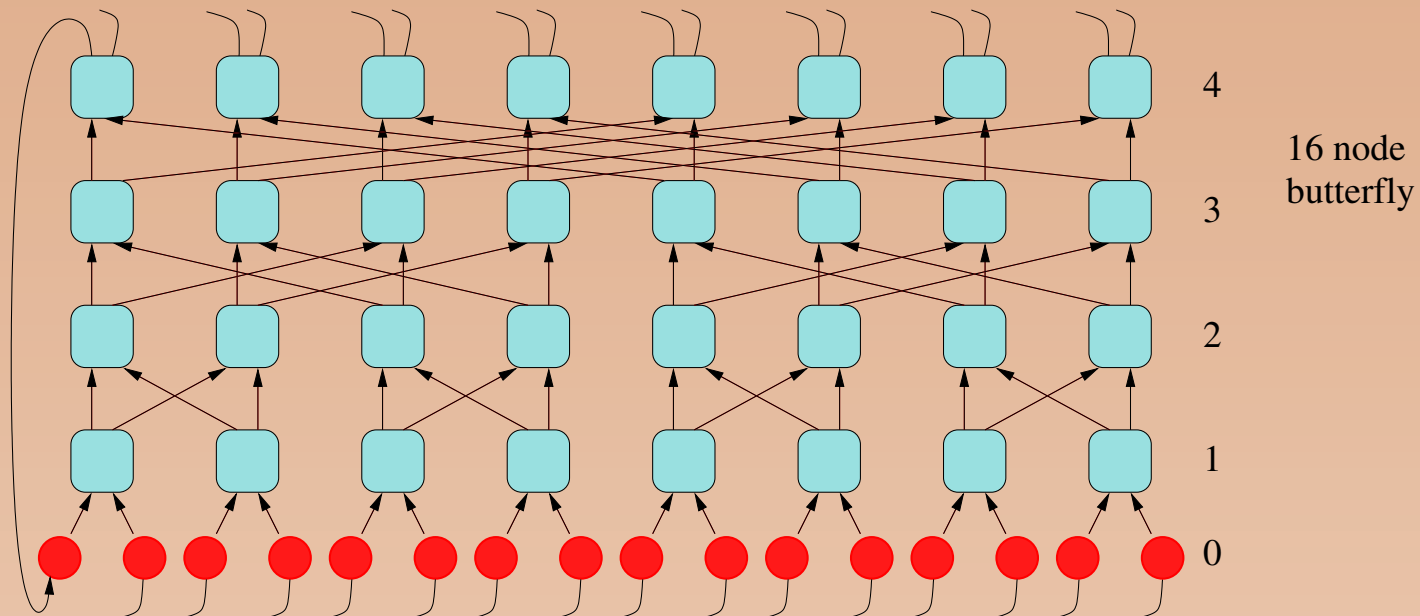
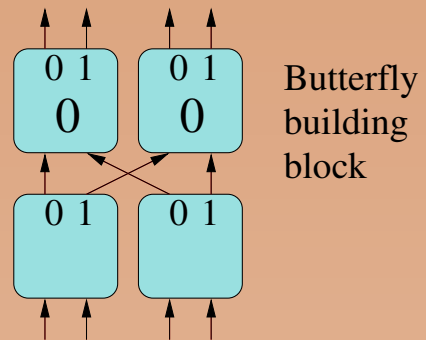
k -ary n -cubes:

number of nodes N	$=$	k^d
switch degree	$=$	$d + 2$
diameter	$=$	$d(k - 1)$
distance	\sim	$d\frac{1}{2}(k - 1)$
network cost	$=$	$O(N)$
total bandwidth	$=$	$2Nb$
bisection bandwidth	$=$	$2k^{(d-1)}b$

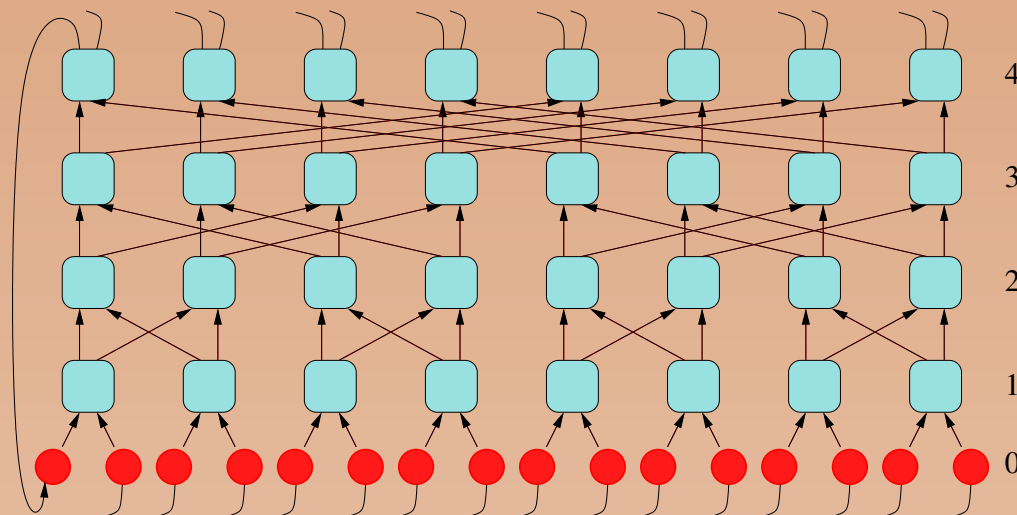
k -ary trees:

number of nodes N	$=$	k^d
number of switches	\sim	k^d
switch degree	$=$	$k + 1$
diameter	$=$	$2d$
distance	\sim	$d + 2$
network cost	$=$	$O(N)$
total bandwidth	$=$	$2 \cdot 2(N - 1)b$
bisection bandwidth	$=$	kb

Butterflies



Butterfly Characteristics



number of nodes N	$= 2^d$
number of switches	$= 2^{d-1}d$
switch degree	$= 2$
diameter	$= d + 1$
distance	$= d + 1$
network cost	$= O(Nd)$
total bandwidth	$= 2^d db$
bisection bandwidth	$= \frac{N}{2}b$

k -ary n -Cubes versus k -ary Trees vs Butterflies

	k -ary n -cubes	binary tree	butterfly
cost per node	$O(N)$	$O(N)$	$O(N \log N)$
distance	$\frac{1}{2} \sqrt[d]{N} \log N$	$2 \log N$	$\log N$
links per node	2	2	$\log N$
bisection	$2N^{\frac{d-1}{d}}$	1	$\frac{1}{2}N$
frequency limit of random traffic	$1/(\sqrt[d]{\frac{N}{2}})$	$1/N$	$1/2$

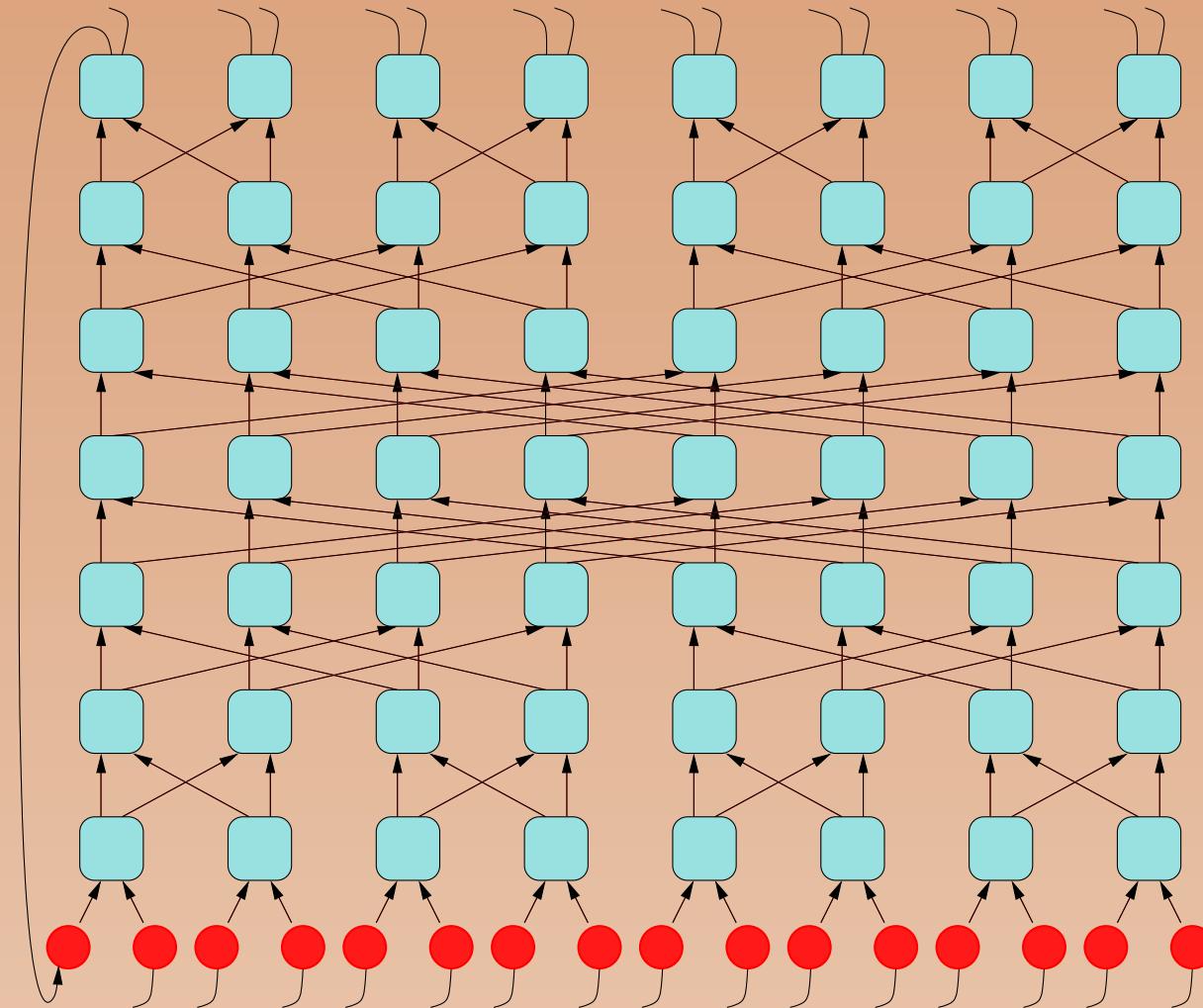


Problems with Butterflies

- Cost of the network
 - ★ $O(N \log N)$
 - ★ 2-d layout is more difficult than for binary trees
 - ★ Number of long wires grows faster than for trees.
- For each source-destination pair there is only one route.
- Each route blocks many other routes.

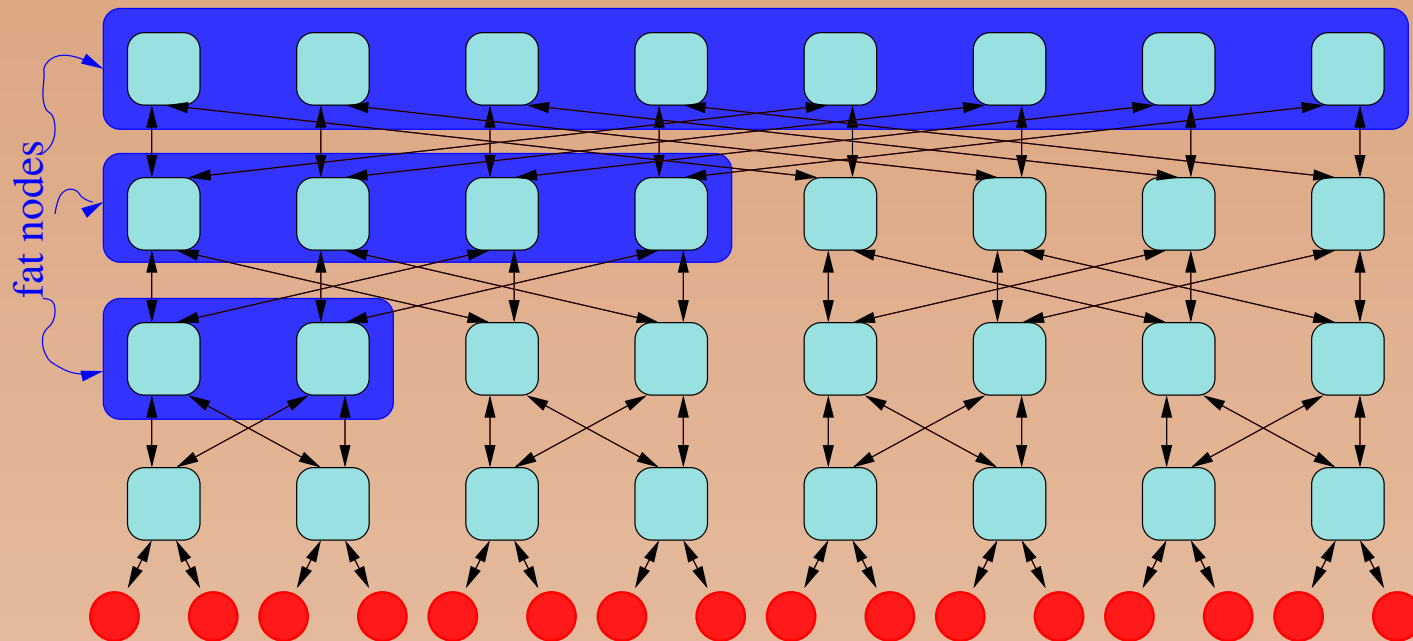


Benes Networks



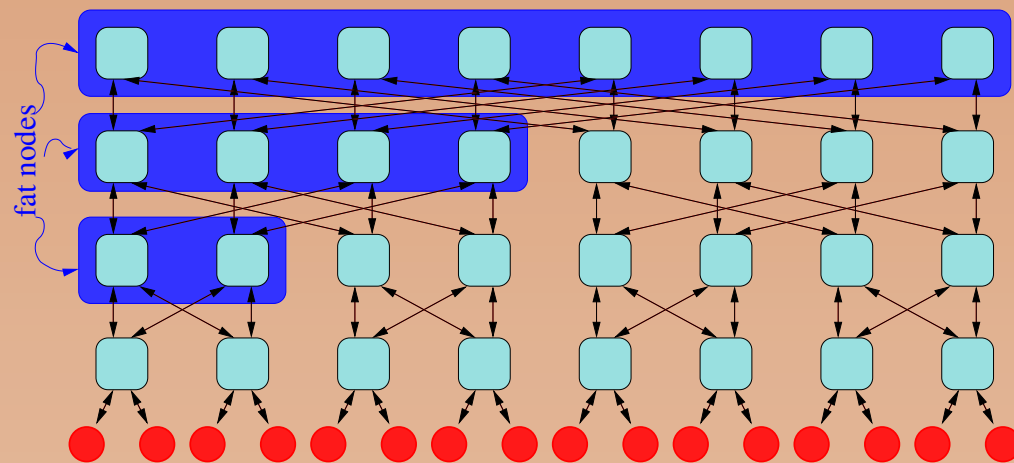
- Many routes;
- Costly to compute non-blocking routes;
- High probability for non-blocking route by randomly selecting an intermediate node [Leighton, 1992];

Fat Trees



16-node 2-ary fat-tree

k -ary n -dimensional Fat Tree Characteristics



16-node 2-ary fat-tree

number of nodes N	$=$	k^d
number of switches	$=$	$k^{d-1}d$
switch degree	$=$	$2k$
diameter	$=$	$2d$
distance	\sim	d
network cost	$=$	$O(Nd)$
total bandwidth	$=$	$2k^d db$
bisection bandwidth	$=$	$2k^{d-1}b$

k -ary n -Cubes versus k -ary d -dimensional Fat Trees

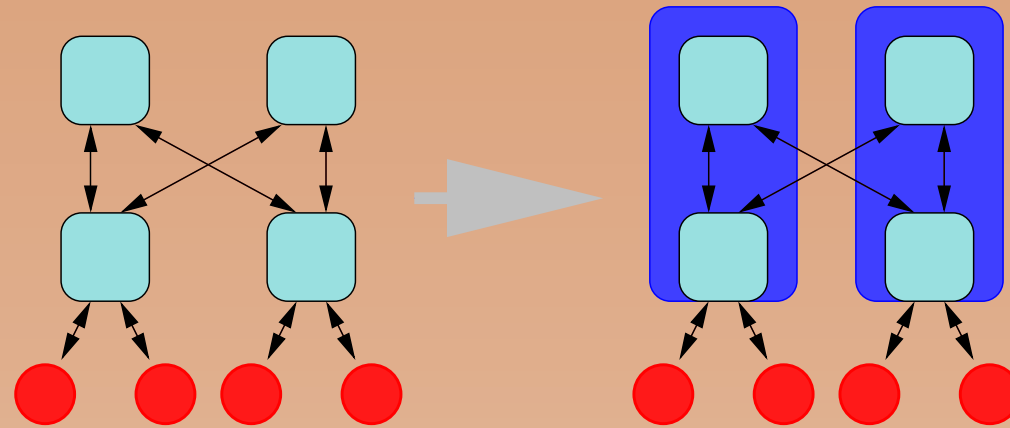
k -ary n -cubes:

number of nodes N	$=$	k^d
switch degree	$=$	d
diameter	$=$	$d(k - 1)$
distance	\sim	$d\frac{1}{2}(k - 1)$
network cost	$=$	$O(N)$
total bandwidth	$=$	$2Nb$
bisection bandwidth	$=$	$2k^{(d-1)}b$

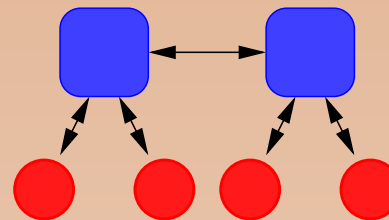
k -ary n -dimensional fat trees:

number of nodes N	$=$	k^d
number of switches	$=$	$k^{d-1}d$
switch degree	$=$	$2k$
diameter	$=$	$2d$
distance	\sim	d
network cost	$=$	$O(Nd)$
total bandwidth	$=$	$2k^d db$
bisection bandwidth	$=$	$2k^{d-1}b$

Relation between Fat Tree and Hypercube

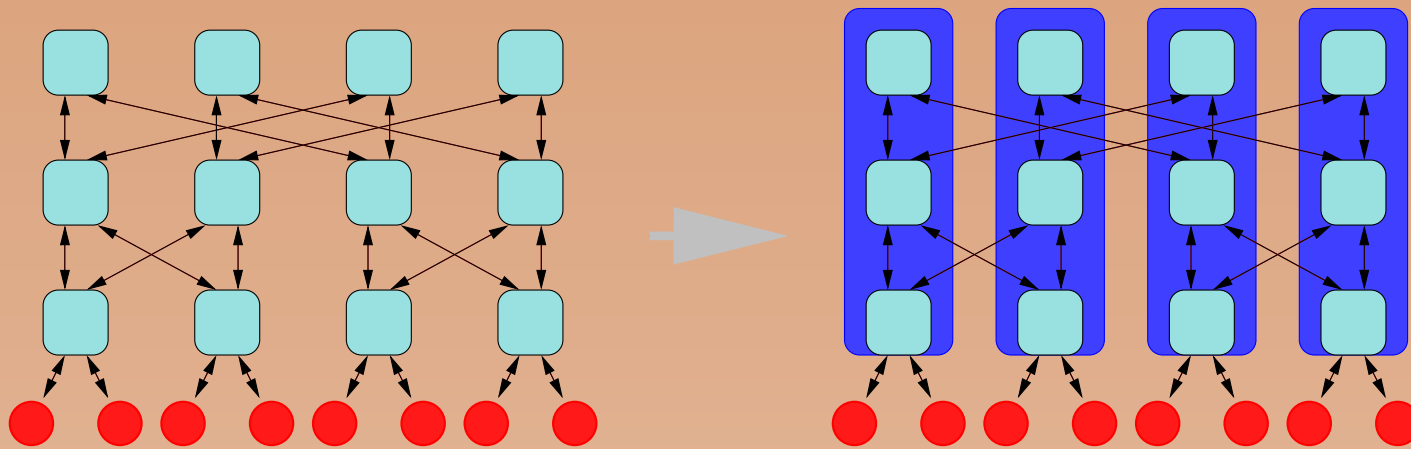


binary 2-dim fat tree

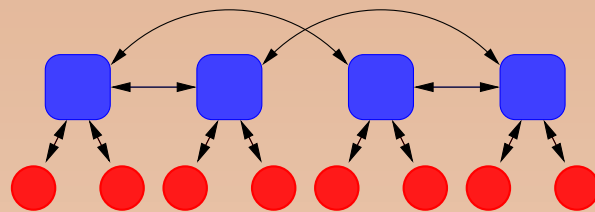


binary 1-cube

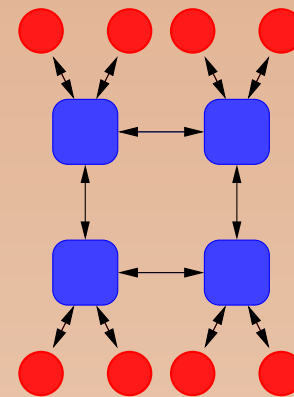
Relation between Fat Tree and Hypercube - cont'd



binary 3-dim fat tree

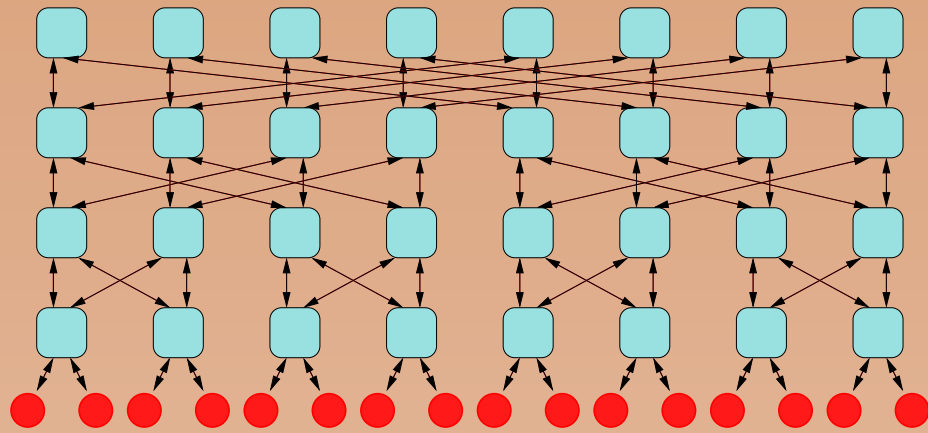


binary 2-cube

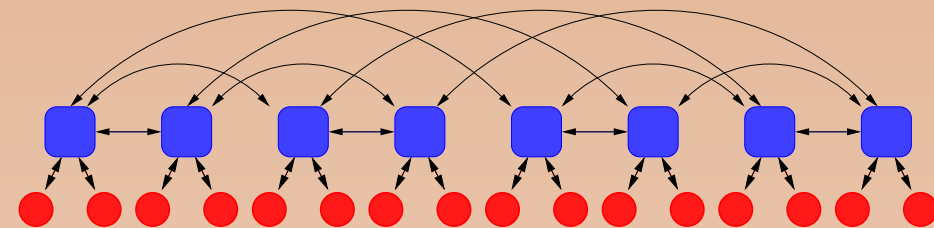
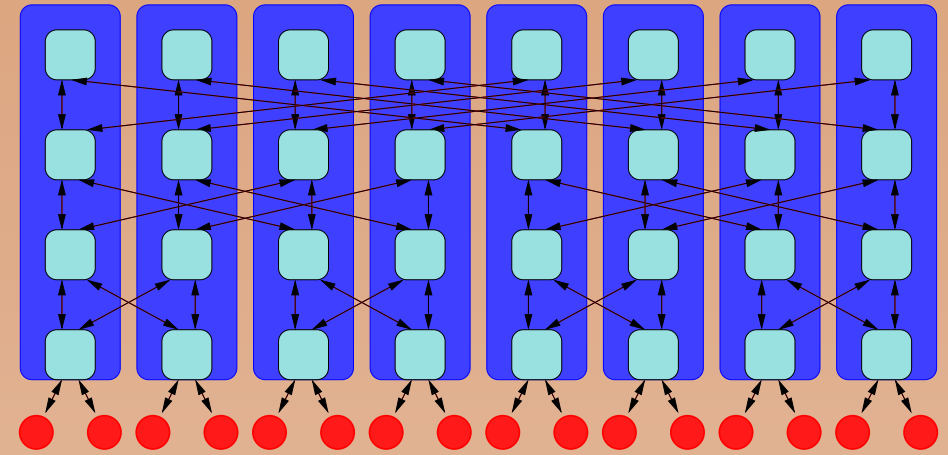


binary 2-cube

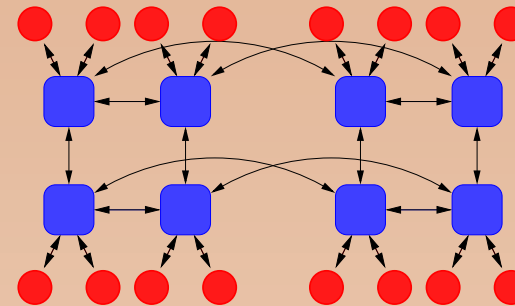
Relation between Fat Tree and Hypercube - cont'd



binary 4-dim fat tree



binary 3-cube



binary 3-cube



Topologies of Parallel Computers

Machine	Topology	Cycle Time [ns]	Channel width [bits]	Routing delay [cycles]	Flit size [bits]
nCUBE/2	Hypercube	25	1	40	32
TMC CM-5	Fat tree	25	4	10	4
IBM SP-2	Banyan	25	8	5	16
Intel Paragon	2D Mesh	11.5	16	2	16
Meiko CS-2	Fat tree	20	8	7	8
Cray T3D	3D Torus	6.67	16	2	16
DASH	Torus	30	16	2	16
J-Machine	3D Mesh	31	8	2	8
Monsoon	Butterfly	20	16	2	16
SGI Origin	Hypercube	2.5	20	16	160
Myricom	Arbitrary	6.25	16	50	16



Trade-offs in Topology Design for the k -ary n -Cube

- Unloaded Latency
- Latency under Load

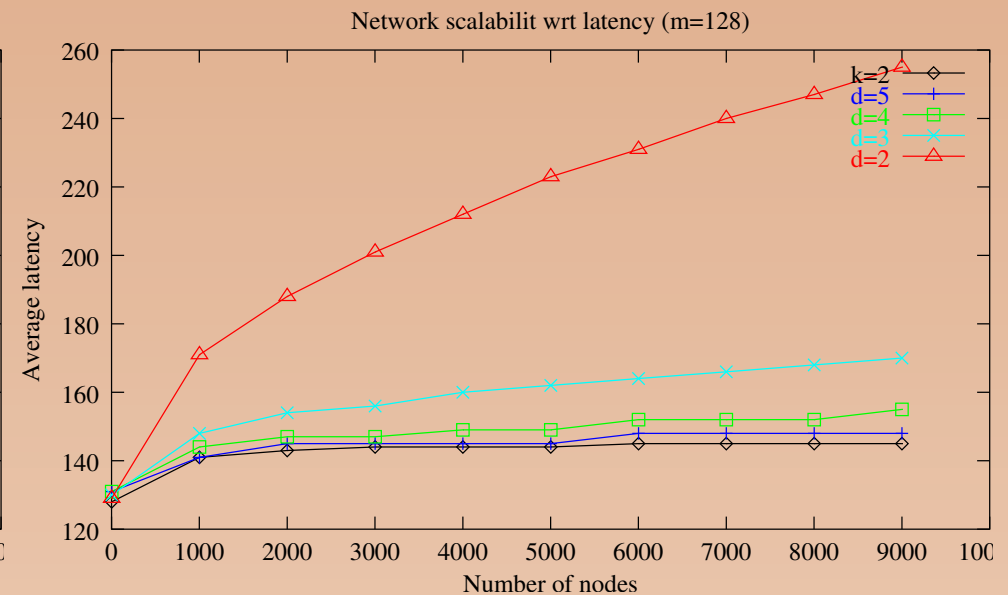
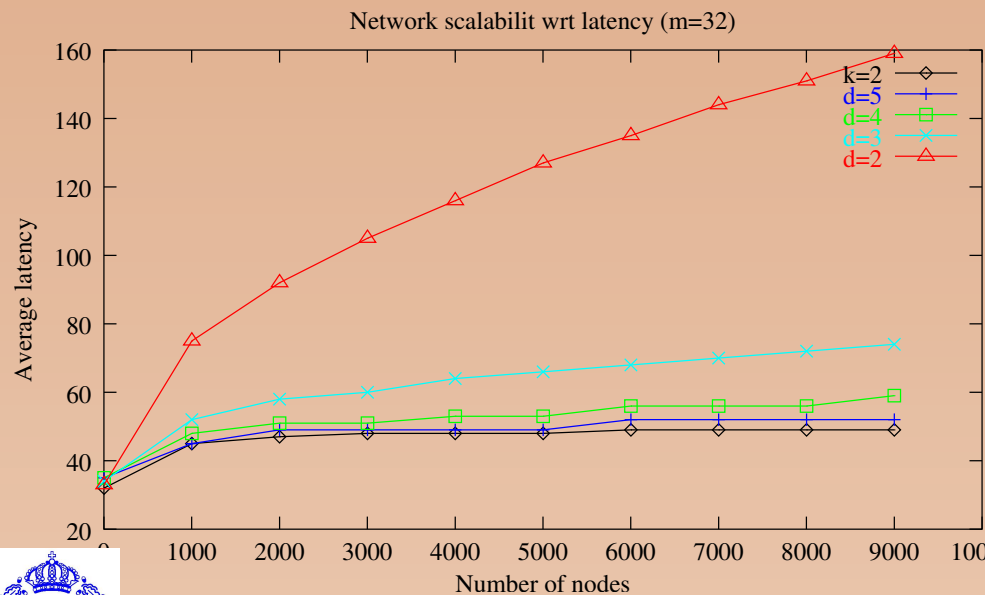


Network Scaling for Unloaded Latency

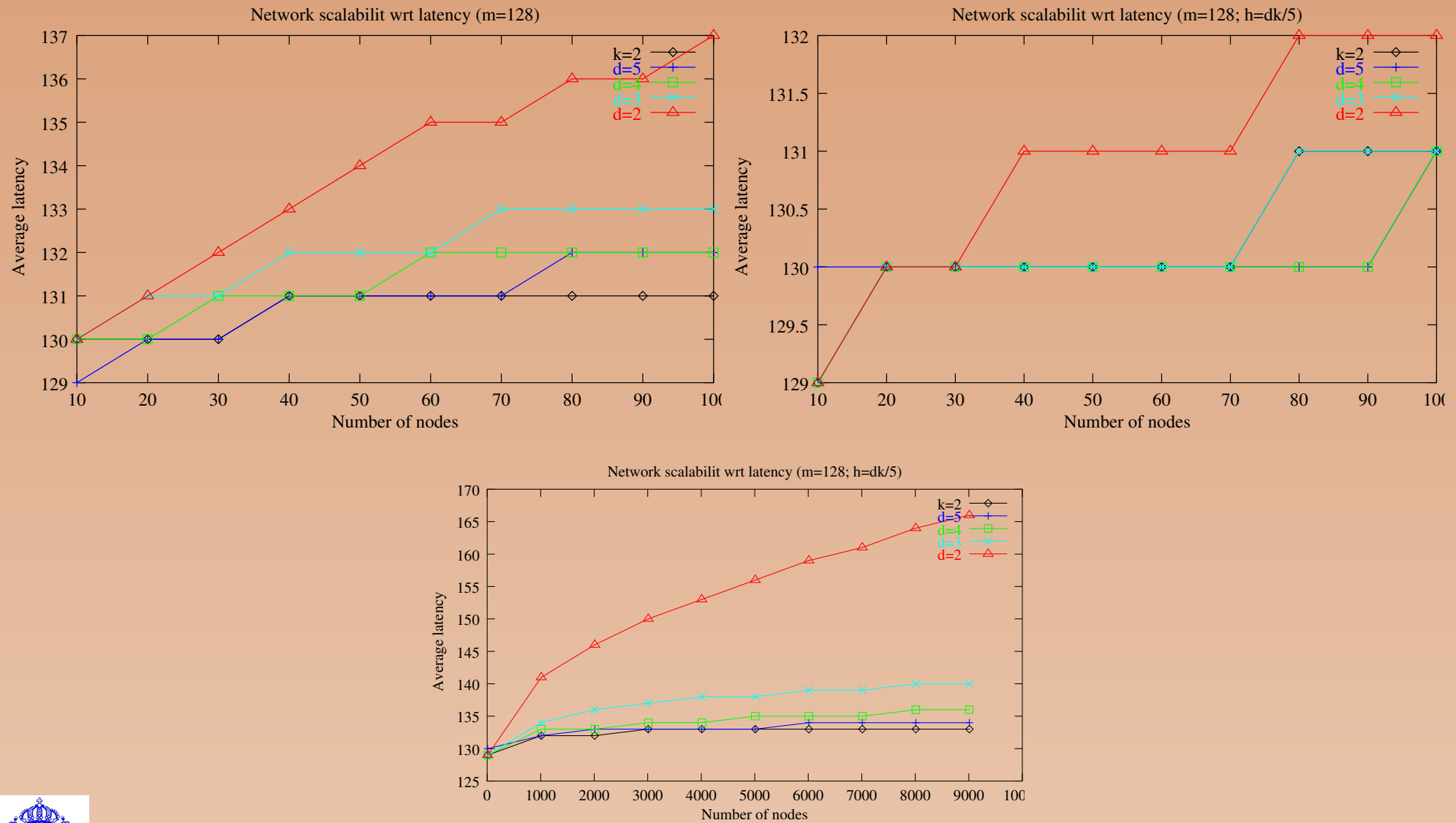
$$\text{Latency}(n) = \text{Admission} + \text{ChannelOccupancy} \\ + \text{RoutingDelay} + \text{ContentionDelay}$$

$$\text{RoutingDelay } T_{ct}(n, h) = \frac{n}{b} + h\Delta$$

$$\text{RoutingDistance } h = \frac{1}{2}d(k-1) = \frac{1}{2}(k-1)\log_k N = \frac{1}{2}(d\sqrt[d]{N} - 1)$$

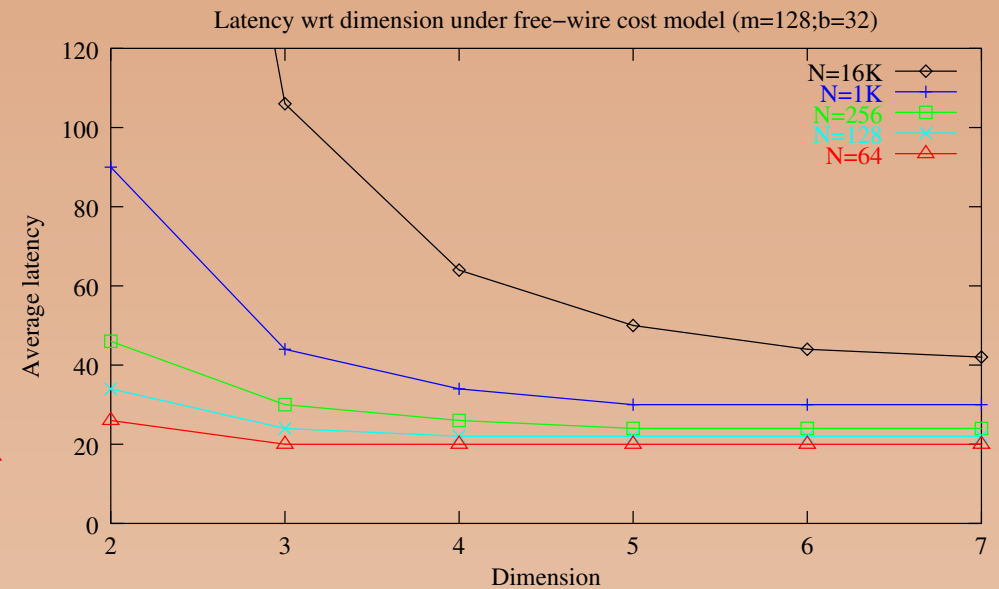
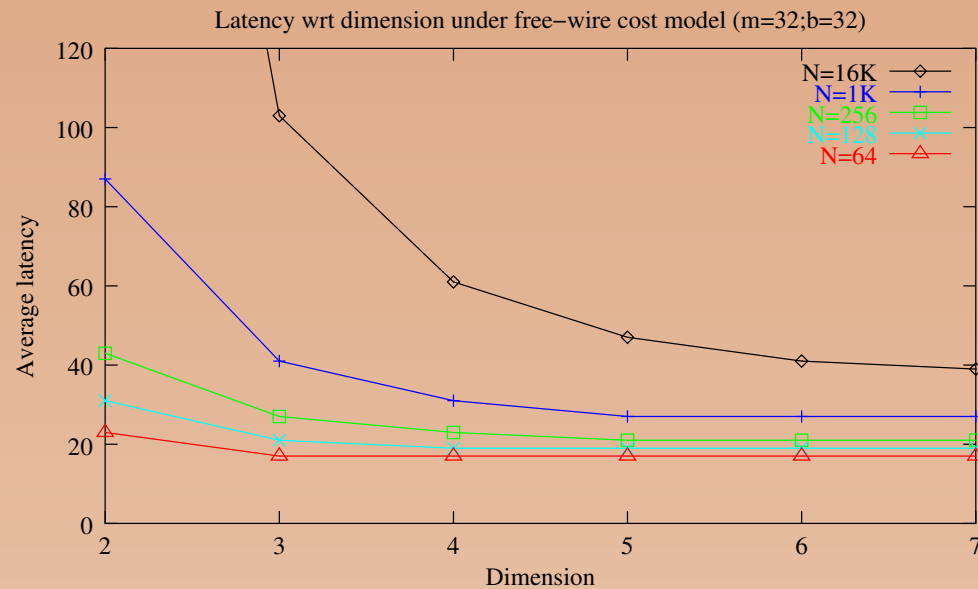


Unloaded Latency for Small Networks and Local Traffic



Unloaded Latency under a Free-Wire Cost Model

Free-wire cost model: Wires are free and can be added without penalty.

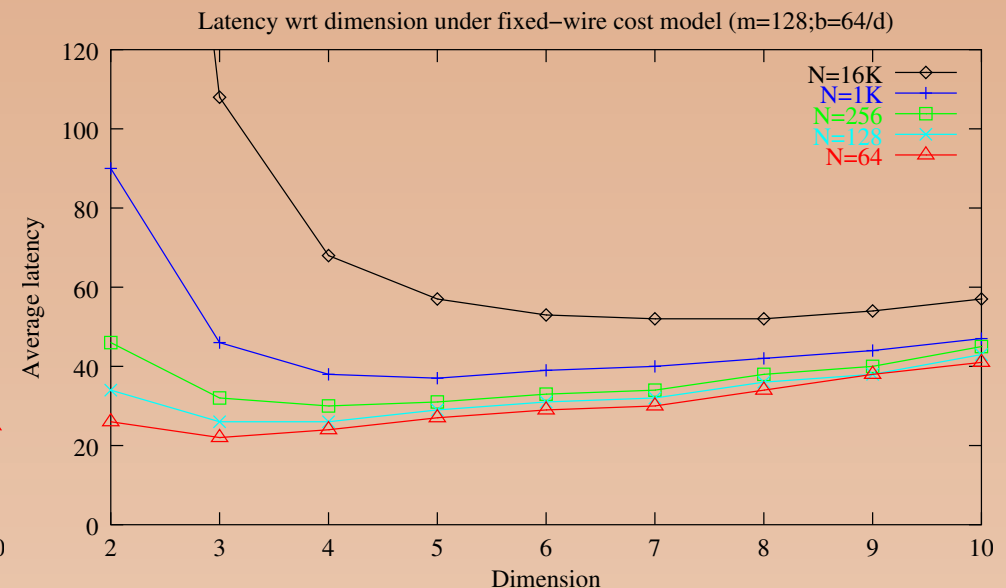
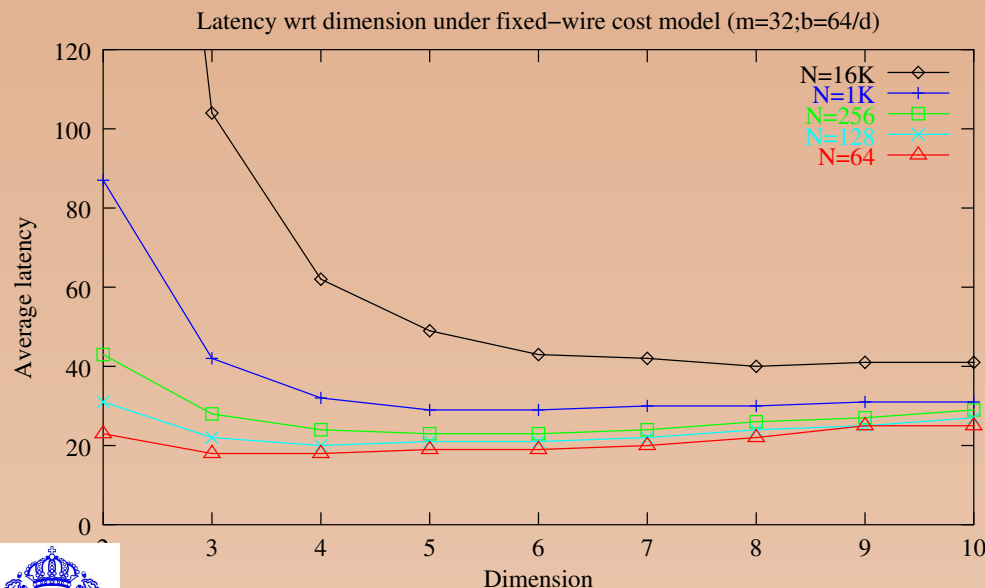


Unloaded Latency under a Fixed-Wire Cost Models

Fixed-wire cost model: The number of wires is constant per node:

128 wires per node: $w(d) = \lfloor \frac{64}{d} \rfloor$.

d	2	3	4	5	6	7	8	9	10
$w(d)$	32	21	16	12	10	9	8	7	6



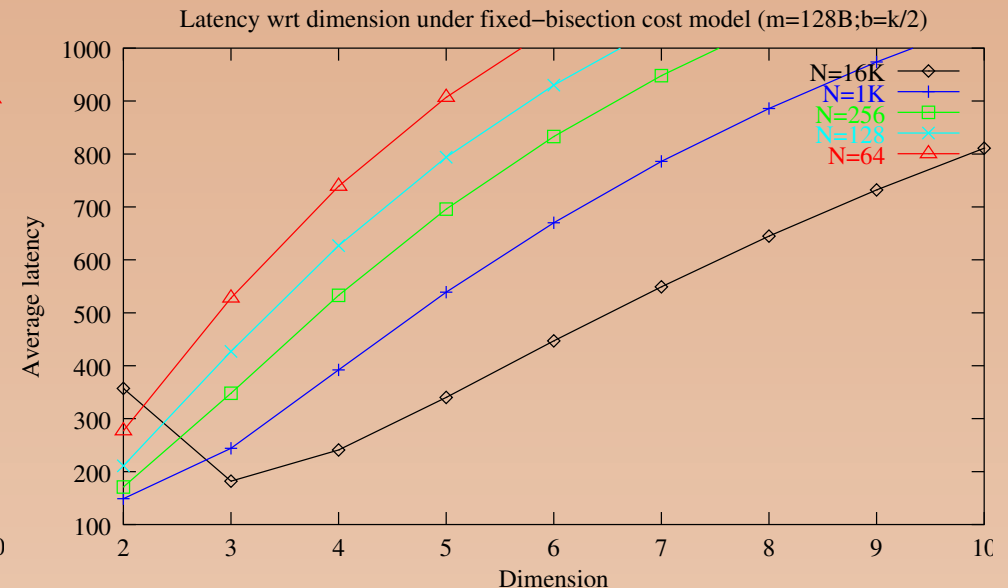
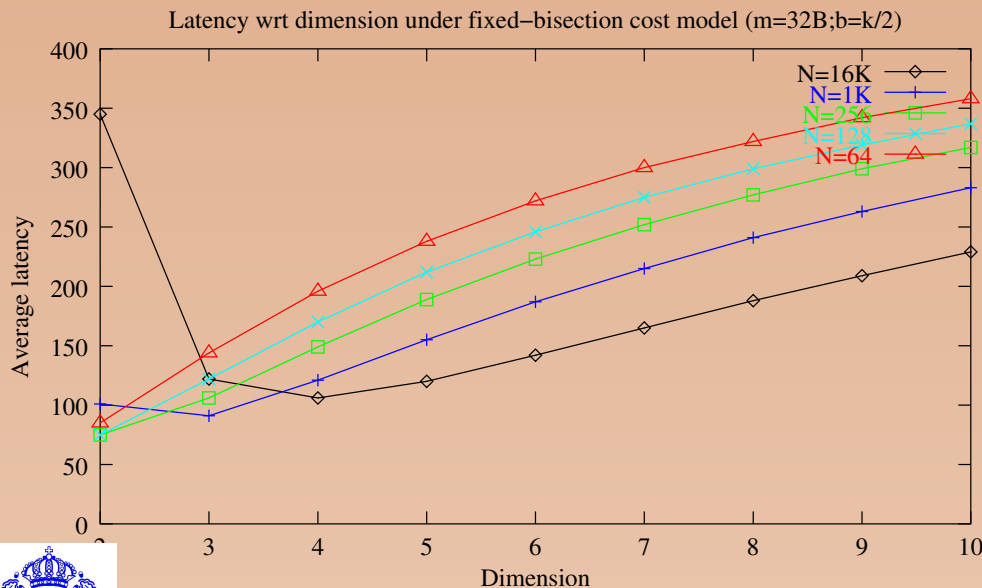
Unloaded Latency under a Fixed-Bisection Cost Models

Fixed-bisection cost model: The number of wires across the bisection is constant:

$$\text{bisection} = 1024 \text{ wires: } w(d) = \frac{k}{2} = \frac{\sqrt[d]{N}}{2}.$$

Example: $N=1024$:

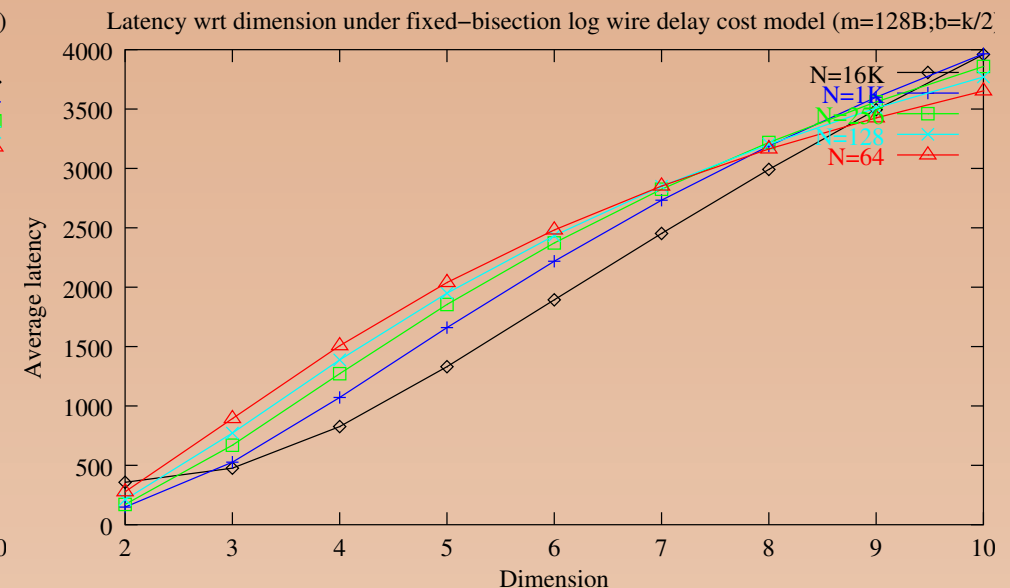
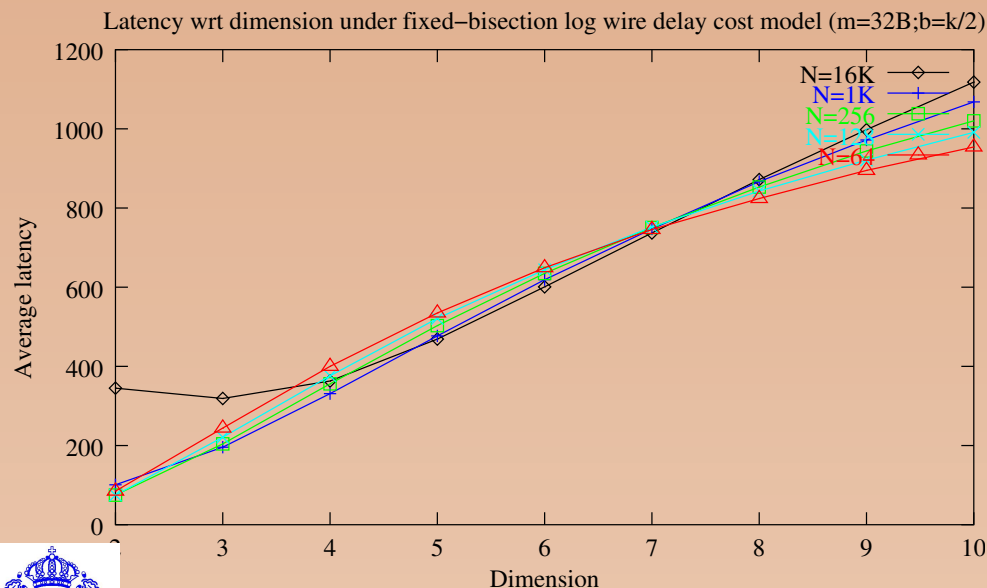
d	2	3	4	5	6	7	8	9	10
$w(d)$	512	16	5	3	2	2	1	1	1



Unloaded Latency under a Logarithmic Wire Delay Cost Models

Fixed-bisection Logarithmic Wire Delay cost model: The number of wires across the bisection is constant and the delay on wires increases logarithmically with the length [Dally, 1990]:
Length of long wires: $l = k^{\frac{n}{2}-1}$

$$T_c \propto 1 + \log l = 1 + \left(\frac{d}{2} - 1\right) \log k$$

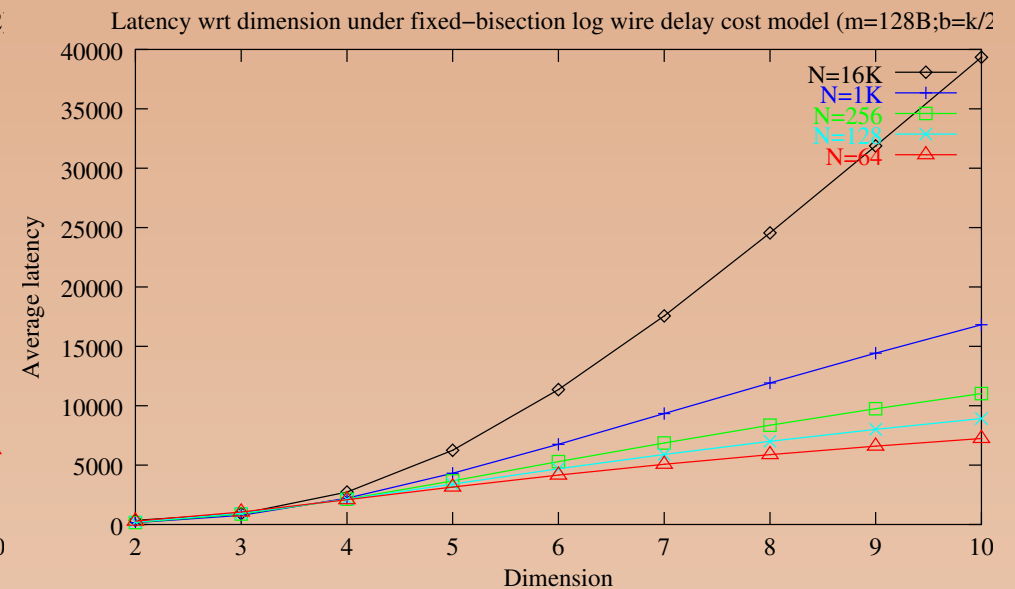
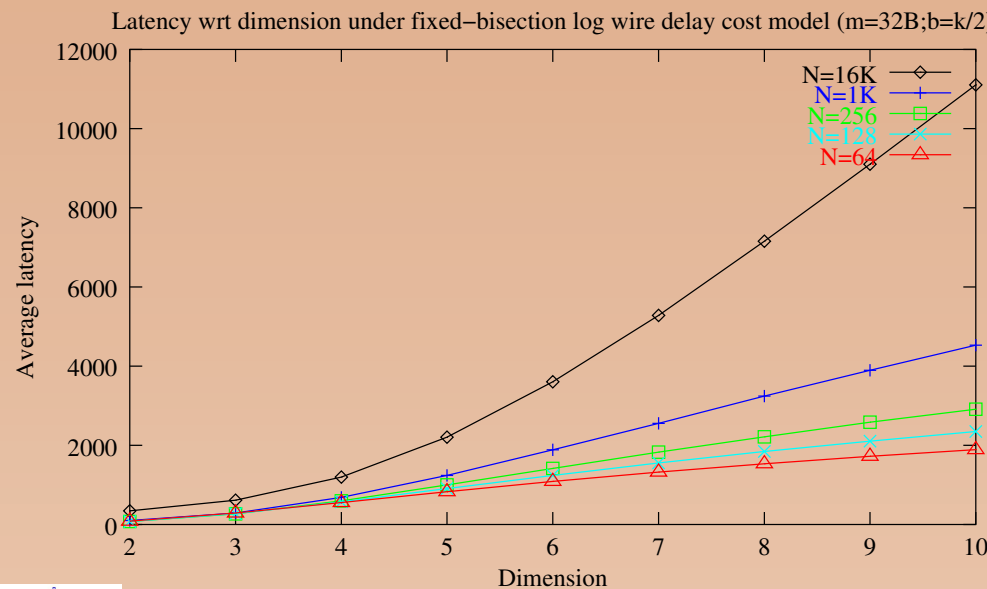


Unloaded Latency under a Linear Wire Delay Cost Models

Fixed-bisection Linear Wire Delay cost model: The number of wires across the bisection is constant and the delay on wires increases linearly with the length [Dally, 1990]:

Length of long wires: $l = k^{\frac{n}{2}-1}$

$$T_c \propto l = k^{\frac{d}{2}-1}$$



Latency under Load

Assumptions [Agarwal, 1991]:

- k -ary n -cubes
- random traffic
- dimension-order cut-through routing
- unbounded internal buffers (to ignore flow control and deadlock issues)



Latency under Load - cont'd

$$\text{Latency}(n) = \text{Admission} + \text{ChannelOccupancy} + \text{RoutingDelay} + \text{ContentionDelay}$$

$$T(m, k, d, w, \rho) = \text{RoutingDelay} + \text{ContentionDelay}$$

$$T(m, k, d, w, \rho) = \frac{m}{w} + dh_k(\Delta + W(m, k, d, w, \rho))$$

$$W(m, k, d, w, \rho) = \frac{m}{w} \cdot \frac{\rho}{(1 - \rho)} \cdot \frac{h_k - 1}{h_k^2} \cdot \left(1 + \frac{1}{d}\right)$$

$$h = \frac{1}{2}d(k - 1)$$

m ... message size

w ... bandwidth of link

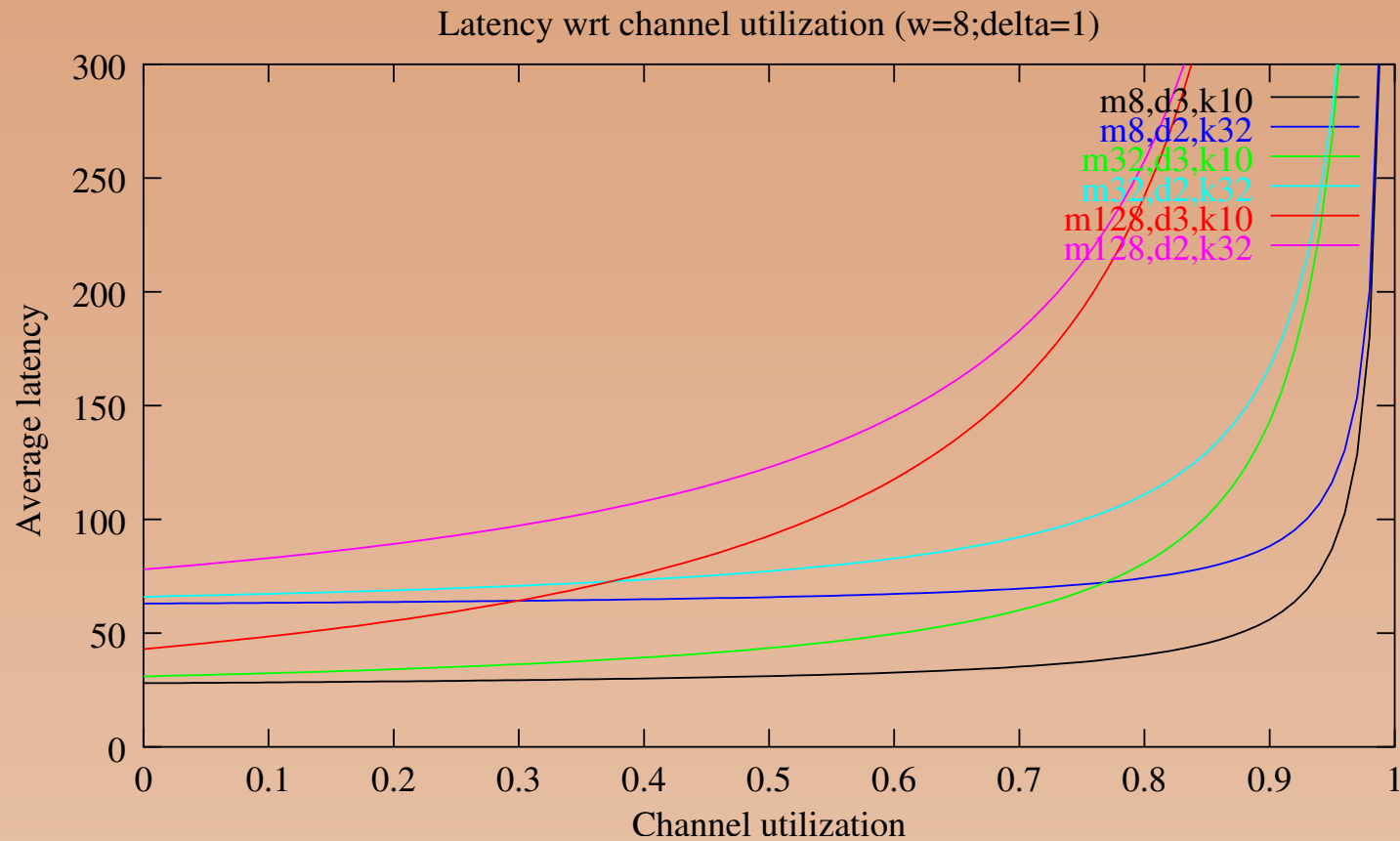
ρ ... aggregate channel utilization

h_k ... average distance in each dimension

Δ ... switching time in cycles



Latency vs Channel Load



Routing

Deterministic routing The route is determined solely by source and destination locations.

Arithmetic routing The destination address of the incoming packet is compared with the address of the switch and the packet is routed accordingly. (relative or absolute addresses)

Source based routing The source determines the route and builds a header with one directive for each switch. The switches strip off the top directive.

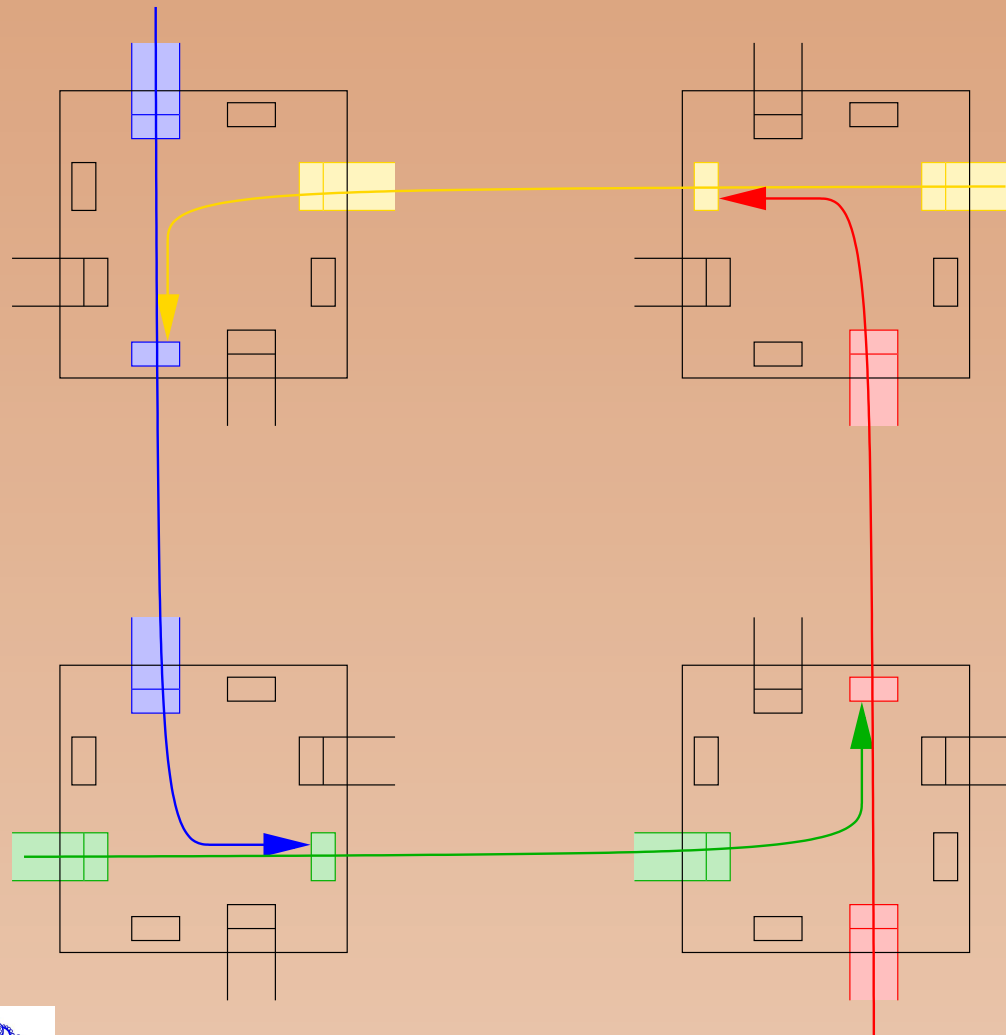
Table-driven routing Switches have routing tables, which can be configured.

Adaptive routing The route can be adapted by the switches to balance the load.

Minimal routing allows only shortest paths while non-minimal routing allows even longer paths.



Deadlock



Deadlock Two or several packets mutually block each other and wait for resources, which can never be free.

Livelock A packet keeps moving through the network but never reaches its destination.

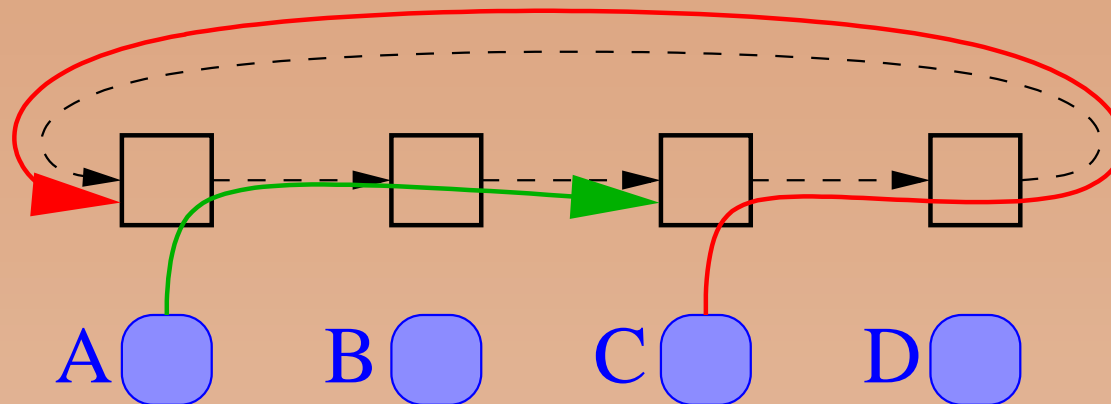
Starvation A packet never gets a resource because it always loses the competition for that resource (fairness).

Deadlock Situations

- Head-on deadlock;
- Nodes stop receiving packets;
- Contention for switch buffers can occur with store-and-forward, virtual-cut-through and wormhole routing. Wormhole routing is particularly sensible.
- Cannot occur in butterflies;
- Cannot occur in trees or fat trees if upward and downward channels are independent;
- Dimension order routing is deadlock free on k -ary n -arrays but not on tori with any $n \geq 1$.



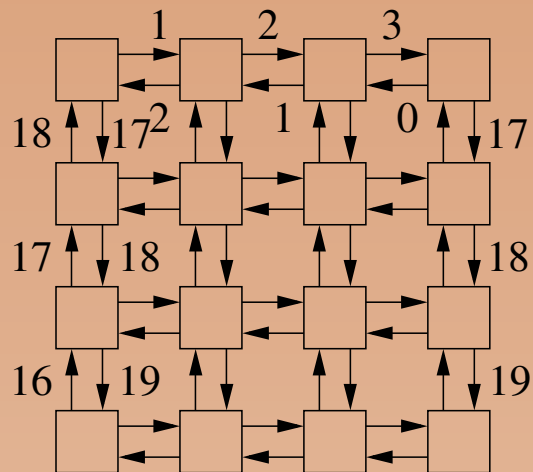
Deadlock in a 1-dimensional Torus



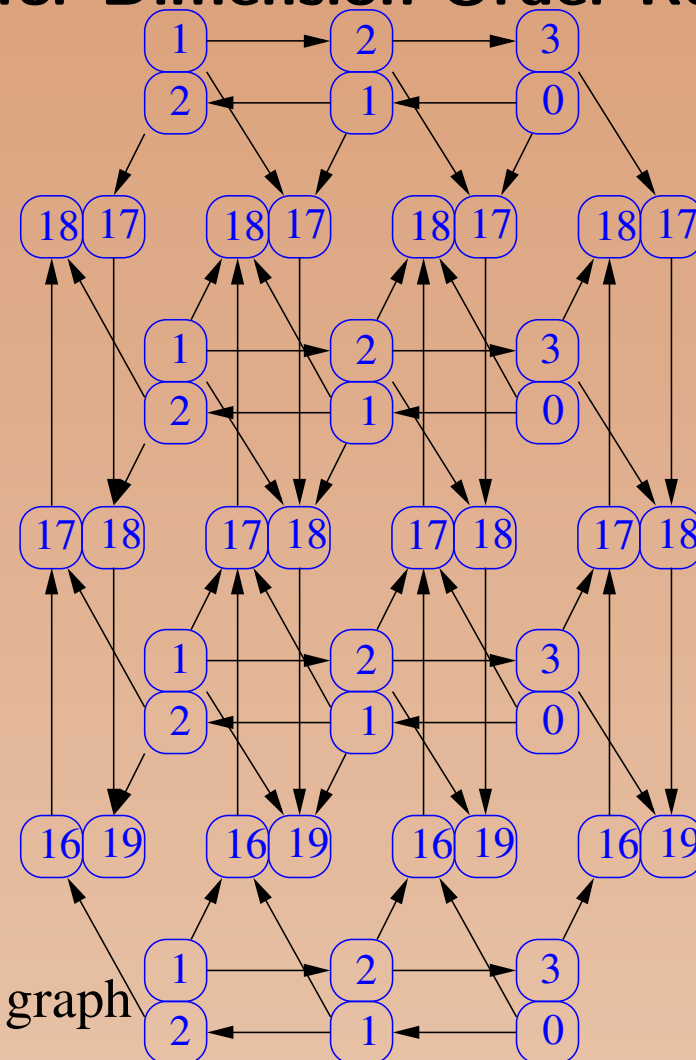
Message 1 from C \rightarrow B, 10 flits

Message 2 from A \rightarrow D, 10 flits

Channel Dependence Graph for Dimension Order Routing



4-ary 2-array



channel dependence graph

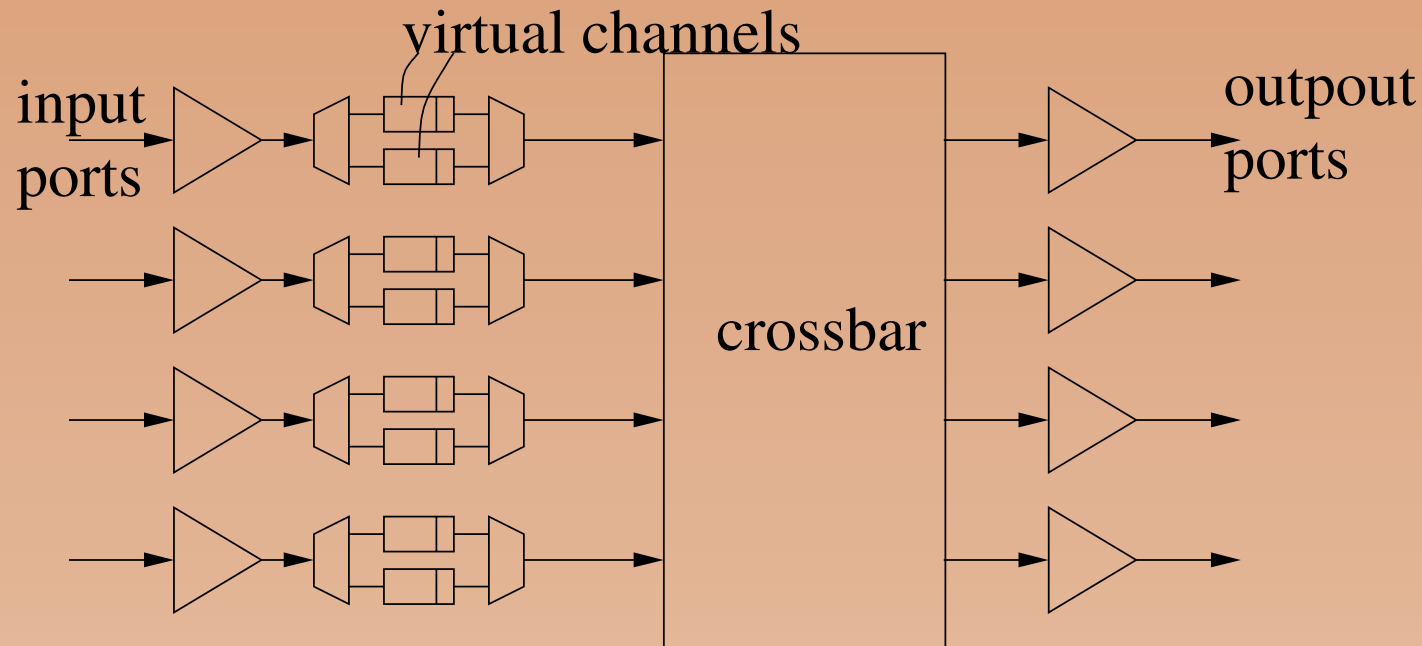
Routing is deadlock free if the channel dependence graph has no cycles.

Deadlock-free Routing

- Two main approaches:
 - ★ Restrict the legal routes;
 - ★ Restrict how resources are allocated;
- Number the channel cleverly
- Construct the channel dependence graph
- Prove that all legal routes follow a strictly increasing path in the channel dependence graph.

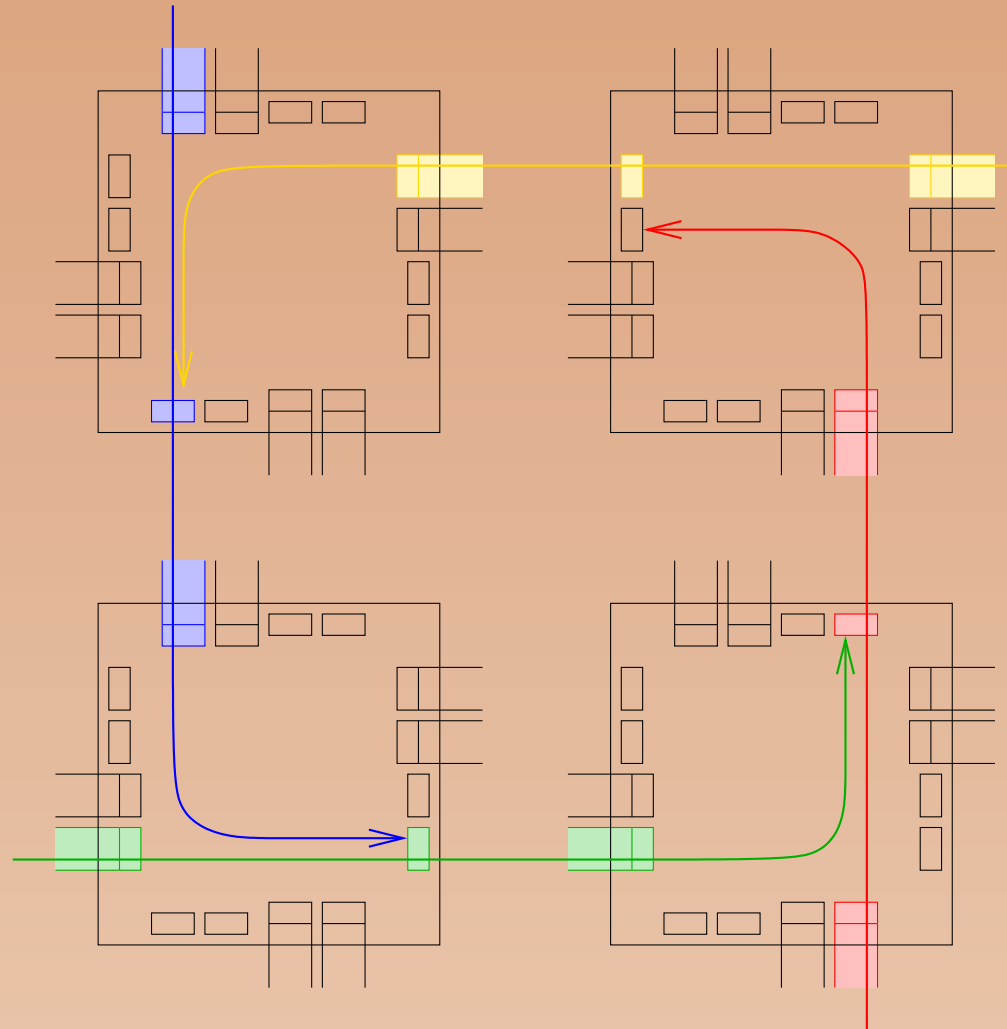


Virtual Channels



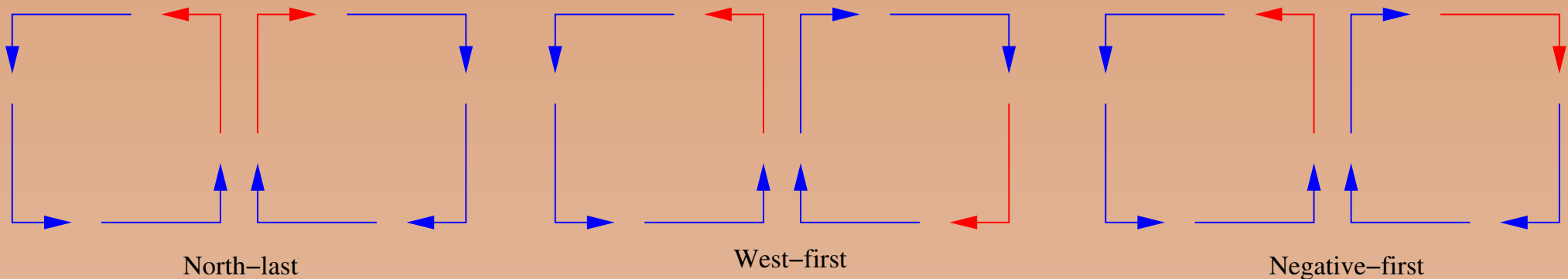
- Virtual channels can be used to break cycles in the dependence graph.
- E.g. all n -dimensional tori can be made deadlock free under dimension-order routing by assigning all wrap-around paths to a different virtual channel than other links.

Virtual Channels and Deadlocks



Turn-Model Routing

What are the minimal routing restrictions to make routing deadlock free?



- Three minimal routing restriction schemes:
 - ★ **North-last**
 - ★ **West-first**
 - ★ **Negative-first**
- Allow complex, non-minimal adaptive routes.
- Unidirectional k -ary n -cubes still need virtual channels.



Adaptive Routing

- The switch makes routing decisions based on the load.
- Fully adaptive routing allows all shortest paths.
- Partial adaptive routing allows only a subset of the shortest path.
- Non-minimal adaptive routing allows also non-minimal paths.
- Hot-potato routing is non-minimal adaptive routing without packet buffering.



Summary

- Communication Performance: bandwidth, unloaded latency, loaded latency
- Organizational Structure: NI, switch, link
- Topologies: wire space and delay domination favors low dimension topologies;
- Routing: deterministic vs source based vs adaptive routing; deadlock;



Issues beyond the Scope of this Lecture

- Switch: Buffering; output scheduling; flow control;
- Flow control: Link level and end-to-end control;
- Power
- Clocking
- Faults and reliability
- Memory architecture and I/O
- Application specific communication patterns
- Services offered to applications; Quality of service



NoC Research Projects

- Nostrum at KTH
- Æthereal at Philips Research
- Proteo at Tampere University of Technology
- SPIN at UPMC/LIP6 in Paris
- XPipes at Bologna U
- Octagon at ST and UC San Diego



To Probe Further - Books and Classic Papers

- [Agarwal, 1991] Agarwal, A. (1991). Limit on interconnection performance. *IEEE Transactions on Parallel and Distributed Systems*, 4(6):613–624.
- [Culler et al., 1999] Culler, D. E., Singh, J. P., and Gupta, A. (1999). *Parallel Computer Architecture - A Hardware/Software Approach*. Morgan Kaufman Publishers.
- [Dally, 1990] Dally, W. J. (1990). Performance analysis of k-ary n-cube interconnection networks. *IEEE Transactions on Computers*, 39(6):775–785.
- [Duato et al., 1998] Duato, J., Yalamanchili, S., and Ni, L. (1998). *Interconnection Networks - An Engineering Approach*. Computer Society Press, Los Alamitos, California.
- [Leighton, 1992] Leighton, F. T. (1992). *Introduction to Parallel Algorithms and Architectures*. Morgan Kaufmann, San Francisco.

